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ABSTRACT

Children's understanding of mathematical concepts, written symbolization of these concepts, and a specifically defined readiness factor were investigated. Thirty-eight first graders were classified as ready or not-ready according to scores on a readiness test. Students were paired, with 11 pairs of not-ready and 8 pairs of ready students; one member of each pair was assigned to an immediate symbolization group and the other to a delayed symbolization group. All students received 12 weeks of instruction on addition and subtraction, with either simultaneous or 5-week-delayed symbolization for ready students, or delayed-until-ready symbolization for not-ready students. The posttest measured ability to interpret, produce, and state answers to number sentences. Scores of not-ready students in the delayed symbolization group were significantly higher on interpretation (a=.05) and production (a=.10) sections than scores of not-ready students in the immediate symbolization group. No significant differences were found on the answer section, nor were significant differences found between any scores of ready students. (MS)

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An Investigation of Oral Language Factors in Readiness for the Written Symbolization of Addition and Subtraction

Katherine B. Hamrick

Pmdc

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FOREWORD

Ed Begle recently remarked that curricular efforts during the 1960's taught us a great deal about how to teach better mathematics, but very little about how to teach mathematics better. The mathematician will, quite likely, agree with both parts of this statement. The layman, the parent, and the elementary school teacher, however, question the thesis that the "new math" was really better than the "old math." At best, the fruits of the mathematics curriculum "revolution" were not sweet. Many judge them to be bitter.

While some viewed the curricular changes of the 1960's to be "revolutionary," others disagreed. Thomas C. O'Brien of Southern Illinois University at Edwardsville recently wrote, "We have not made any fundamental change in school mathematics."[1] He cites Allendoerfer who suggested that a curriculum which heeds the ways in which young children learn mathematics is needed. Such a curriculum would be based on the understanding of children's thinking and learning. It is one thing, however, to recognize that a conceptual model for mathematics curriculum is sound and necessary and to ask that the child's thinking and learning processes be heeded, it is quite another to translate these ideas into a curriculum which can be used effectively by the ordinary elementary school teacher working in the ordinary elementary school classroom.

Moreover, to propose that children's thinking processes should serve as a basis for curriculum development is to presuppose that curriculum makers agree on what these processes are. This is not the case, but even if it were, curriculum makers do not agree on the implications which the understanding of these thinking processes would have for curriculum development.

In the real world of today's elementary school classroom, where not much hope for drastic changes for the better can be foreseen, it appears that in order to build a realistic, yet sound basis for the mathematics curriculum, children's mathematical thinking must be studied intensively in their usual school habitat. Given an opportunity to think freely, children clearly display certain patterns of thought as they deal with ordinary mathematical situations encountered daily in their classroom. A videotaped record of the outward manifestations of a child's thinking, uninfluenced by any teaching on the part of the interviewer, provides a rich source for conjectures as to what this thinking is, what mental structures the child has developed, and how the child uses these structures when dealing with the ordinary concepts of arithmetic. In addition, an intensive analysis of this videotape generates some conjectures as to the possible sources of what adults view as children's "misconceptions" and about how the school environment (the teacher and the materials) "fights" the child's natural thought processes.

The Project for the Mathematical Development of Children (PMDC)[2] set out to create a more extensive and reliable basis on which to build mathematics curriculum. Accordingly, the emphasis in the first phase is to try to understand the children's intellectual pursuits, specifically their attempts to acquire some basic mathematical skills and concepts.

The PMDC, in its initial phase, works with children in grades 1 and 2. These grades seem to comprise the crucial years for the development of bases for the future learning of mathematics, since key mathematical concepts begin to form at these grade levels. The children's mathematical development is studied by means of:

1. One-to-one videotaped interviews subsequently analyzed by various individuals.

2. Teaching experiments in which specific variables are observed in a group teaching setting with five to fourteen children.

3. Intensive observations of children in their regular classroom setting.

4. Studies designed to investigate intensively the effect of a particular variable or medium on communicating mathematics to young children.

[1]"Why Teach Mathematics?" The Elementary School Journal 73 (Feb. 1973), 258-68.

[2]PMDC is supported by the National Science Foundation, Grant No. PES 74-18106-A03.

5. Formal testing, both group and one-to-one, designed to provide further insights into young children's mathematical knowledge.

The PMDC staff and the Advisory Board wish to report the Project's activities and findings to all who are interested in mathematical education. One means for accomplishing this is the PMDC publication program.

Many individuals contributed to the activities of PMDC. Its Advisory Board members are: Edward Begle, Edgar Edwards, Walter Dick, Renee Henry, John LeBlanc, Gerald Rising, Charles Smock, Stephen Willoughby and Lauren Woodby. The principal investigators are: Merlyn Behr, Tom Denmark, Stanley Erlwanger, Janice Flake, Larry Hatfield, William McKillip, Eugene D. Nichols, Leonard Pikaart, Leslie Steffe, and the Evaluator, Ray Carry. A special recognition for this publication is given to the PMDC Publications Committee, consisting of Merlyn Behr (Chairman), Thomas Cooney and Tom Denmark.

Eugene D. Nichols
Director of PMDC

PREFACE

This publication is intended to share with the reader an investigation of the relationship between first grade children's understanding of written mathematical symbols and a specifically defined "readiness" factor based on verbal facility with the concepts to be symbolized. The investigation was a dissertation study submitted to the faculty of the University of Georgia, 1976.

The data from the PMDC Fall 1975 Testing Program were used in this investigation, and the results of this investigation were given to the PMDC principal investigators for possible use in their studies.

The author of this study in indebted to the following PMDC personnel for their advice and support throughout the study: Merlyn Behr, Tom Cooney, Tom Denmark, Larry Hatfield, Eugene Nichols, and Leslie Steffe. A special debt of gratitude is owed to the author's major professor, William McKillip, a principal investigator for PMDC. Thanks are also due Janelle Hardy and Maria Pitner for handling the publication of this dissertation.

AN INVESTIGATION OF ORAL LANGUAGE FACTORS IN READINESS FOR THE WRITTEN SYMBOLIZATION OF ADDITION AND SUBTRACTION

by

ANNA KATHERINE BARR HAMRICK

B.S.Ed., The University of Georgia 1969

M.Ed., The University of Georgia 1974

A Dissertation Submitted to the Graduate Faculty
of the University of Georgia in Partial Fulfillment
of the
Requirements for the Degree
DOCTOR OF EDUCATION

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1976

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AN INVESTIGATION OF ORAL LANGUAGE FACTORS IN
READINESS FOR THE WRITTEN SYMBOLIZATION
OF ADDITION AND SUBTRACTION

by

ANNA KATHERINE BARR HAMRICK

Approved:

William D. McKillip Date 7/9/76
Major Professor

Louis W. Wilson Date 7/7/76
Chairman, Reading Committee

Approved:

Dean, Graduate School

July 13, 1976
Date

ANNA KATHERINE BARR HAMRICK
An Investigation of Oral Factors in Readiness for the Written Symbolization of Addition and Subtraction.
(Under the direction of WILLIAM D. MCKILLIP)

This study investigated children's understanding of mathematical concepts, written symbolization of these concepts, and a specifically defined "readiness" factor. This readiness factor was defined as follows:

> Given a topic in elementary mathematics, there are sets of objectives, which indicate mastery of the topic. Omitting those objectives concerned with reading, writing, and speed of response, a child is ready for the introduction of the symbolization of the topic when he has mastered the objectives of the topic verbally, perhaps with the aid of pictures or manipulatives.

This study is based on the assumption that written mathematical symbols are similar to written language symbols. When a student is learning either type of symbol, he must associate the symbol to a meaning. The language symbols are first associated with known sound symbols, and the sound symbols arouse meaning in the mind of the child. It is possible that written arithmetic symbols should also first be associated with known sound symbols. In other words, verbal facility may be a readiness factor for learning the written mathematical symbols.

The subjects were 38 first grade students at Barrow Elementary School, Athens, Georgia. In September, 1975, the subjects were classified as ready or not ready according to scores on a readiness test based on the definition of readiness. The subjects were paired by means

of the readiness test scores and Key Math Test scores. This resulted in eleven pairs of not ready subjects and eight pairs of ready subjects. One member of each pair was randomly assigned to an immediate symbolization group and the other member assigned to a delayed symbolization group.

All subjects received 12 weeks of instruction in introductory addition and subtraction. The immediate symbolization groups of both ready and not ready subjects experienced a treatment in which written symbolization was introduced simultaneously with the concepts. The delayed symbolization of ready subjects experienced a treatment in which written symbolization was delayed for five weeks. The delayed symbolization group of not ready subjects experienced a treatment in which written symbolization was delayed until each subject was judged to be ready on the basis of the above definition.

A posttest was designed to measure the subject's ability to interpret, produce, and state answers to number sentences. A student's meaningful learning of the symbolization of addition and subtraction was defined in terms of the subject's ability to produce, interpret, and state answers to number sentences. Thus the posttest was a measure of a subject's meaningful learning of written symbolization.

The scores of the not ready subjects in the delayed symbolization group were significantly higher, $\alpha = .05$, on the interpretation section of the posttest and were significantly higher, $\alpha = .10$, on the production section of the posttest then the scores of the not ready subjects in the immediate symbolization group. There were no significant differences between the scores of these two groups on the answer section or between any of the posttest scores of ready subjects.

Conclusions

(1) Children's readiness, as defined above, does not affect ready student's meaningful learning of the symbolization of addition and subtraction.

(2) A delay of symbolization may cause ready students to become bored, thus affecting the efficiency of learning.

(3) Children's readiness, as defined above, affects not ready students meaningful learning of the symbolization of addition and subtraction. The learning is more meaningful if the symbolization is delayed until the students are ready.

(4) The delay of symbolization for not ready students facilitated learning.

Index words: Readiness, Symbolization, Oral Language, Addition, Subtraction.

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ACKNOWLEDGMENTS

The writer wishes to express her appreciation to the people who helped in the investigation and preparation of this dissertation.

A special debt of gratitude is owed to the writer's major professor, Dr. William D. McKillip. Dr. McKillip provided invaluable support and guidance for the planning of the study, the investigation, and the writing of the dissertation. Without the help of Dr. McKillip, this study would not have been possible.

Sincere appreciation is expressed to the members of the reading committee, Dr. James Wilson, Dr. Mary Ann Byrne, Dr. Thomas Cooney, and Dr. Roy O'Donnell, for their suggestions and encouragement.

The teaching experiment was made possible by the cooperation of the principal, the first grade teachers, and the students at David C. Barrow Elementary School. The writer is also indebted to Mrs. Nancy Wein for her help in the instructional part of the study.

Finally, the writer thanks her husband, Gayle, and her daughter, Beth, for their encouragement and understanding throughout the study.

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Chapter I

THE PROBLEM

The purpose of this study was to investigate the relationship among children's understanding of mathematical concepts, written symbolization of these concepts, and a well defined "readiness" for written symbolization based on verbal facility with the concepts to be symbolized. It was hypothesized that readiness for written symbolization, as defined, would influence the course of learning and the success of instruction. In order to test this general hypothesis, a study was designed at the first grade level utilizing the learning of addition and subtraction concepts for small whole numbers and the symbols that express them. There are a number of factors which must be discussed in presenting the rationale of the present study.

Prior to the organization of the study, the investigator examined errors children make in elementary mathematics. In particular, errors were noticed in which children seemed to be manipulating symbols according to their own rules. The investigator felt that while some of the errors children make in elementary mathematics are a result of lack of understanding of the mathematics itself, others seem to be the result of confusion associated with the written symbolization of the mathematics. This source of error could be seen in mistakes children made when working with the written forms of problems they could correctly

solve when the problems were done verbally. It is possible that the written symbolization did not have associated with it the meanings utilized by the children when working verbally.

A child's readiness to learn a topic should always be considered before the introduction of the topic. However, a child's readiness for learning the written symbolization of the topic, not readiness for learning the topic, was the main focus of the present study. Readiness can be defined in terms of a child's prerequisite learning, his maturational stage, and his motivation to learn. The present study was designed to investigate readiness defined by hypothesized prerequisites for learning the written symbolization of a topic.

The learning of the written symbolization of mathematics is similar in many respects to the learning of the written symbolization of language, that is, learning to read. In reading and language education, verbal facility is considered to be an important readiness factor; a child is not considered ready to read until he has an adequate speaking and hearing knowledge of the words and sentences he is expected to read. However, there has been little consideration of the readiness factor of a spoken vocabulary in relation to the mathematical symbols a child is expected to read. It is possible that many children are introduced to the symbolization of mathematics before they have an adequate speaking and hearing vocabulary in mathematics.

ERRORS -- CONTRASTS BETWEEN VERBAL AND WRITTEN WORK

There are examples in children's work that illustrate the contrast between verbal and written work. The following examples were taken from a test administered to first grade students at Barrow School

during the Spring of 1975.[1] Four items on the test required the students to produce an appropriate number sentence for a picture of a set or sets of objects. Two items required the students to write a number sentence and two items required the students to say a number sentence. Of the 38 students tested, 10 students gave correct number sentences for all four items and 17 students gave incorrect number sentences for all four items. The remaining 11 students said correct number sentences for one or two of the items requiring the student to say a number sentence but wrote incorrect number sentences for all items requiring the student to write a number sentence.

For example, when given the first picture in Figure 1 and asked to write the number sentence, one child wrote "43." However, when given a similar picture, the second picture in Figure 1, and asked to say the number sentence, the same child said, "Five and two is seven."

Another child, when given the first picture in Figure 2 and asked to write the number sentence wrote, "4 - 2 = 4." However, the same child when given a similar picture, the second picture in Figure 2, and asked to say the number sentence said, "Seven take away two makes five." Both children could give the correct answer verbally, but answered incorrectly in writing.

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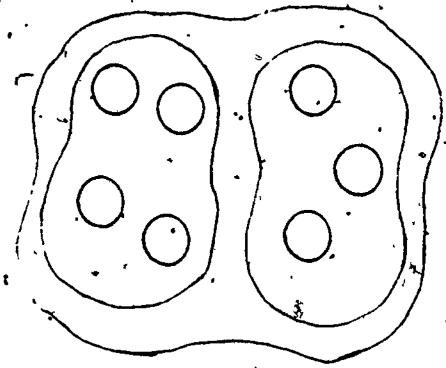
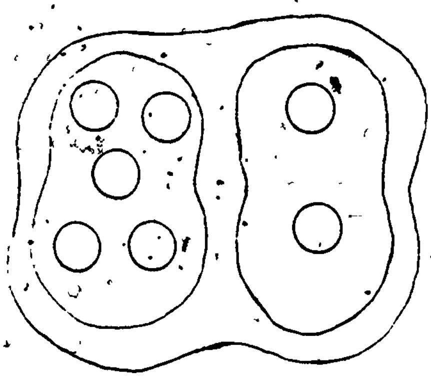
[1]William D. McKillip, "First Grade Verbal and Manipulative Mode Study 1974-1975" (Project for the Mathematical Development of Children, University of Georgia, Athens, Georgia) report of Principal Investigator, November, 1975. (Mimeographed.)

Picture 1



The child was asked to write the number sentence. The child wrote, "43".

Picture 2

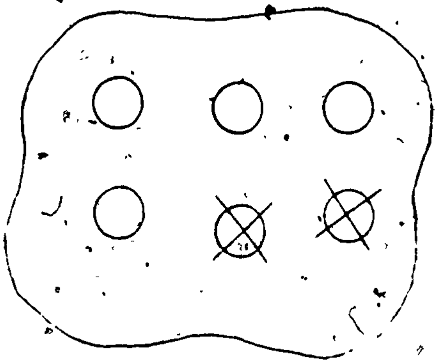


The child was then asked to say the number sentence. The child said, "Five and two is seven."

Figure 1

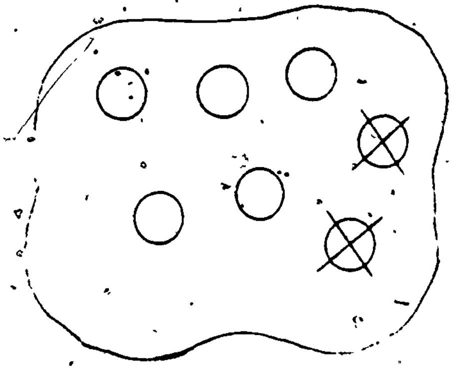
First Grade Verbal and Manipulative Mode Study 1974-1975; Responses of one Child to Picture Stimuli

Picture 1.



The child was asked to write the number sentence. The child wrote, "4 - 2 = 4."

Picture 2.



The child was then asked to say the number sentence. The child said, "Seven take away two makes five."

Figure 2

First Grade Verbal and Manipulative Mode Study
1974-1975; Responses of One Child
to Picture Stimuli

MEANING

Since the children in this study were learning the symbolization of addition and subtraction for the first time, it was important that the learning be meaningful. In the thirty-seventh yearbook of the National Council of Teachers of Mathematics, Folsom stated:

> In the early stages of learning, a primary goal is to help the child move from physical world situations to the abstract ideas of operating on whole numbers. . . . Although the child does determine sums, proficiency in computing is not the goal of early instruction.[2]

In this study, the definition of meaning of the symbolization of addition and subtraction is a modification of the definition used by Fordham[3] who conducted an investigation of the nature of meaning. Paraphrasing Fordham, the meaning of an arithmetic symbol or group of symbols is a pairing in the mind of an individual of the symbol, or symbols, with some action that is appropriate for the symbol. The pairing in the mind can be accomplished in one of two ways. The individual could be given the symbol and he then must interpret the symbol as referring to an action. On the other hand, the individual could be shown an action, or told a description of an action, and he then must produce the symbol or symbols that describe the action.

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[2]Mary Folsom, "Operations on Whole Numbers", Mathematics Learning in Early Childhood, ed, Joseph N. Payne, Thirty-seventh Yearbook of the National Council of Teachers of Mathematics (Reston: National Council of Teachers of Mathematics, 1975), p. 162.

[3]Dennis L. Fordham, "An Investigation of Third, Fourth, and Fifth Graders' Knowledge of Meanings of Selected Symbols Associated with Multiplication of Whole Numbers" (unpublished Doctoral dissertation, The University of Georgia, 1974), pp. 7-8.

In order for a group of symbols representing a concept or operation to be meaningful to a child, the concept or operation itself must first be meaningful to the child. A concept or operation is meaningful to a child when the child can pair the concept or operation to an appropriate action. For example, the operation of addition is assumed to be meaningful to a child if the child can pair, in an interpretative sense, a question such as, "How much are three and three?" to the action of the joining of two sets or to an act such as counting. The child also demonstrates that addition is meaningful to him by producing the operation that is appropriate for the action, i.e., the child might respond to the union of two sets by saying, "Three and three are six."

The symbols of addition are meaningful to a child if the child can pair symbols such as "3 + 3 = ___" to the union of two sets or an act such as counting. Again the pairing in the child's mind can be demonstrated by interpreting the symbols or by producing the symbols.

It is possible, as illustrated by the contrast between children's verbal and written work, for the verbal representation of the operation or concept to be meaningful to a child, while the child may lack meaning for the written symbolization of the operation or concept. For example, as previously described in Figure 2, one child was presented with a picture illustrating the number sentence, "6 - 2 = 4" and the child wrote, "4 - 2 = 4." However, the same child when presented with a similar picture and asked to say the number sentence answered correctly. The child demonstrated that the verbal representation of the operation of subtraction was meaningful to him, while the written symbols of subtraction were not.

READINESS

Readiness is a general concept which has been applied to a child's preparedness to begin any learning task. Much of the work in readiness has been in the related fields of language and reading education. Downing and Thackray define readiness for learning as

> . . . the stage firstly when the child can learn easily and without emotional strain, and secondly, when the child can learn profitably because efforts at teaching give gratifying results.[4]

The ex post facto nature of this and other similar definitions of readiness prevent the use of the definitions to assess a child's readiness for learning a topic before the topic is introduced.

A more workable description of readiness is that a child's readiness for learning a topic is a function of the child's maturation, motivation, and preparation. The child's physical maturation, his motivation to learn, and his preparation in terms of mastery of prerequisite learnings are assessable before the introduction of a topic. In the present study, emphasis will be on a child's preparation in terms of mastery of prerequisite learnings.

In reading and language education, the linguistic ability of the child is usually considered to be an important factor for learning to read. Bond stated that, "Language facility is one of the more important

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[4]John Downing and D. V. Thackray, Reading Readiness (London: University of London Press Ltd., 1975), p. 9.

readiness factors that are definitely trainable."[5] Tinker and McCullough state, "The greater the ability to comprehend material presented in oral form, the greater the proficiency in the use of oral language, the more ready the child will be for beginning reading."[6]

When the relationship between the symbols of language and the symbols of mathematics is considered, it is possible that verbal facility with the language of mathematics or linguistic ability in mathematics is a readiness factor for the meaningful learning of the symbols of mathematics.

AN ANALOGY TO READING

Similarities between reading and mathematics were summarized by Hickerson who stated:

> Since arithmetic is a system of symbolism just as language is a system of symbolism why shouldn't the accepted principles underlying the understanding and use of language symbols apply to the understanding and use of arithmetic symbols? It is the writer's conviction that they should apply.[7]

Hickerson also emphasized the importance of verbal facility with language and arithmetic symbols in these analogies from his list of parallel implications for teaching:

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[5]Guy L. Bond and others, Pre-primers, Three of Us, Play with Us, Fun with Us, with Teacher's Guide. (Chicago: Lyons & Carnahan, 1954), p. 18.

[6]Miles A. Tinker and Constance M. McCullough, Teaching Elementary Reading (New Jersey: Prentice-Hall Inc., 1975), p. 97.

[7]J. Allen Hickerson, "Similarities Between Teaching Language and Arithmetic", The Arithmetic Teacher, VI (November, 1959), p. 241.

Language is Learned Best When	Arithmetic is Learned Best When
Oral vocabulary and sentence structure are acquired in relation to the learner's experience by listening to and talking about the things experienced;	Oral language is acquired which represents in complete sentence form the the quantitative relation in problem situations;
Written words are read as symbols standing for already known spoken words;	Written arithmetic symbols are introduced as shorthand ways of writing already known spoken words;[8]

Note that the written words or written arithmetic symbols "stand for" or "are introduced as short hand ways of writing" already known spoken words. Thus Hickerson implies that in beginning arithmetic, ~~as in~~ beginning reading, the ability to verbalize the written symbols in complete sentence form should be learned prior to the written symbols. This could be interpreted to mean that verbal facility is a readiness factor that contributes to the understanding of the written symbols of arithmetic.

Theories of language development confirm that verbal or spoken language is primary and that written language is secondary and dependent on verbal language. The linguist Hill states that, ". . . all writing systems are essentially representations of the forms of speech, rather than representations of ideas or objects in the nonlinguistic world."[9] Hill gives several reasons for this assumption. First, the fact that speech is older than writing and second that all of today's communities of men have speech or language but not all have writing.

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[8] Hickerson, p. 243.

[9] Archibald A. Hill, Introduction to Linguistic Structures, From Sound to Sentence in English (New York: Harcourt, Brace & World, Inc., 1958), p. 2.

He also states that, "Written symbols can be understood, furthermore, insofar as they fit into a linguistic structure. . . ."[10] There are animal and insect "languages" that are based on something other than sound; for example the language of bees is based on body movements. Hill states, however, that ". . . no human language is so constructed. Even the manual language of the deaf is derived from the preexistent spoken language of the community."[11]

The results of a research study by Tatham also imply that written language is dependent on spoken language. Tatham investigated whether children read better when the reading material is closely related to the children's oral language. The study was restricted to second and fourth grade children. She found that "a significant number of second and fourth graders comprehend material written with frequent oral language patterns better than material written with infrequent oral language patterns."[12] Tatham concludes that ". . . it is logical and in keeping with linguistic knowledge to use children's patterns of language structure in written material to facilitate learning the concept that spoken and written language are related."[13]

Educators in language and reading education seem to be applying the theory that spoken language is primary and that written symbols are

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[10] Hill, p. 3.

[11] Hill, p. 4.

[12] Susan Masland Tatham, "Reading Comprehension of Materials Written with Select Oral Language Patterns: A Study at Grades Two and Four," Reading Research Quarterly, II (Spring 1970), 423.

[13] Tatham, p. 424.

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based on sound symbols. When describing the teaching of reading, Allen stated:

> If we are concerned with teaching reading, we do not now make the mistake of the teacher of phonics who puts the cart before the horse by beginning with the written letter and talking about sounds as the powers or value of a letter. Instead, by beginning where the child begins, with his already attained control of sounds, we proceed with a systematic association between sound and visual symbol. . . ."[14]

It does not seem to be the case, however, that the relationship between written symbols and oral symbols is used when teaching mathematics. The investigator reviewed two widely used first grade arithmetic texts, the Silver Burdett first grade text[15] and the Addison-Wesley first grade text.[16] In these texts, the symbolization of addition and subtraction is introduced simultaneously with the concepts of addition and subtraction. Some verbalization of number sentences is suggested in the teacher's directions, but it is suggested that this verbalization accompany written symbolic number sentences. It is possible that an adequate oral vocabulary of symbols and sentences in arithmetic is an important readiness factor for the meaningful learning of the written symbolization of arithmetic as well as the written

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[14] Harold B. Allen, "A Pharos for the Institute," The English Language in the School Program, ed. R. F. Hogan (Illinois: National Council of Teachers of English, 1966), p. 5.

[15] John F. LeBlanc and others, Mathematics, Teachers Edition 1, (Morristown: Silver Burdett Company, 1976).

[16] Robert E. Eicholz and others, Elementary School Mathematics, (Menlo Park: Addison-Wesley Publishing Company, 1971).

symbolization of language. It was the purpose of this study to investigate that possibility.

DEFINITIONS OF TERMS

Meaning of the Symbol "a + b = c" or "a - b = c"

A meaning of the symbol "a + b = c," or "a - b = c," is pairing in the mind of an individual of the symbol with some action that is appropriate for the symbol. The appropriate actions are those which lead from a representation of the problem to an answer to the question, "How many are represented by the symbol?" This definition was paraphrased from Fordham[17] with modifications as to the type of operation. An example of an appropriate action for the symbol "4 - 2 = ____" would be to construct a set of four objects and remove a set of two objects and count the remaining objects.

Demonstration of the Meaning of a Symbol

An individual may demonstrate that he knows the meaning of a symbol in one of the following two ways:

(1) Given a symbol or symbols, the individual must interpret the symbol or symbols by describing or carrying out an action that is appropriate for the symbol. For example, to interpret the symbol "6 - 2 = ____," an individual must describe or carry out an action similar to one of the following:

(a) Construct, or draw, a set of six objects and remove a set of two objects and count the remaining objects.

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[17]Fordham, pp. 7-8.

(b) Demonstrate, or draw, six jumps forward on a number line and two jumps backward and report final position.

(c) Say, "Six minus one is five, and one less is four.", or, "Six minus three is three, and one more is four.", etc.

(d) Construct, or draw, a set of six objects and a set of four objects and compare the sets; then state, "Six minus two is four."

(2) Given an action or description of an action which is appropriate for a symbol or symbols, the individual must produce the symbol or symbols. For example, when shown one of the preceding actions, the individual must produce the symbol " 6 - 2 = 4."

Readiness for the Symbolization of a Topic in Elementary Mathematics

Readiness for the symbolization of a topic in elementary mathematics is defined to mean the following: Given a topic in elementary mathematics, there are lists of objectives, the attainment of which indicates mastery of the topic. Omitting those objectives that are concerned with reading, writing, or speed of response, a child is ready for the introduction of written symbolization of the topic when he has mastered the objectives of the topic verbally, perhaps with the aid of pictures or manipulatives.

Readiness for the Symbolization of Addition and Subtraction

The following specific definition of readiness for symbolization of addition and subtraction is based on the objectives used by the

school in which the study was conducted: A child is ready for the symbolization of addition and subtraction when he has mastered the following objectives verbally, perhaps with the aid of pictures or manipulatives:

(1) Given objects or a picture that illustrate the union of two sets, the student states the sum and says the number sentence.

(2) Given a collection of objects, and given verbally an addition number sentence, a + b = c, the student forms sets having a and b elements respectively and illustrates the union of the sets and states the sum.

(3) Given objects or a picture that illustrate the removal or partitioning of a subset, the student states the difference and says the number sentence.

(4) Given a collection of objects and given verbally a subtraction number sentence, a - b = c, the student forms a set having a elements and removes or partitions a subset having b elements and states the difference.

This definition of readiness does not include operations on a number line or missing addend problems because the "topic" as usually presented in the subjects' school did not include these items.

Readiness Test

The readiness test is a test based on the objectives outlined in the definition of readiness for the introduction of addition and subtraction. It was constructed by the investigator and used to classify students in the study as "ready" or "not ready." A child was

classified as ready if the child demonstrated mastery of the items on the readiness test. A child was otherwise classified as not ready. The test is included in Appendix A, and a further explanation of the test and mastery of the items is included in Chapter III.

Posttest

The posttest is a test constructed by the investigator to evaluate the subjects' ability to interpret, produce, and give answers to addition and subtraction number sentences. The test is included in Appendix D and a further explanation of the test is in Chapter III.

Ready-delayed Group

Ready-delayed group, or (RD), refers to the eight first grade subjects who were classified as "ready" on the basis of the readiness test and who experienced a treatment in which written symbolization of addition and subtraction was delayed for five weeks.

Ready-immediate Group

Ready-immediate group, or (RI), refers to the eight first grade subjects who were classified as "ready" on the basis of the readiness test and who experienced a treatment in which written symbolization of addition and subtraction were introduced simultaneously with the concepts.

Not Ready-delayed Group

Not Ready-delayed group, or (ND), refers to the eleven first grade subjects who were classified as "not ready" on the basis of the readiness test and who experienced a treatment in which written

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symbolization of addition and subtraction was delayed until the children were judged ready.

Not Ready-immediate Group

Not Ready-immediate group, or (NI), refers to the eleven first grade subjects who were classified as "not ready" on the basis of the readiness test and who experienced a treatment in which written symbolization of addition and subtraction was introduced simultaneously with the concepts.

STATEMENT OF THE PROBLEM

The purpose of this study was to investigate whether children's readiness as previously defined influences the efficiency of learning and the meaningfulness of learning the written symbolization of addition and subtraction by first grade children. The influence is hypothesized to be in the following way:

(1) Children who <u>have not</u> reached this state should learn more efficiently and the learning should be more meaningful if the symbolization is delayed until the children have reached this state.

(2) Children who <u>have</u> reached this state should learn more efficiently and the learning should be more meaningful if the symbolization is introduced immediately.

STATEMENT OF HYPOTHESES

The definition of the demonstration of meaning given earlier required that the student interpret given number sentences, produce

number sentences when given certain actions on sets, and state answers to open number sentences in both cases. It is assumed that the student who can do all three of the above for a given symbolic number sentence demonstrates that the symbols are more meaningful to him than to a student who can do only one or two of the above. Thus to measure meaningfulness, the posttest was divided into three sections: a section in which the subjects were required to interpret number sentences; a section in which the subjects were required to produce number sentences; and a section in which the subjects were required to produce answers to open number sentences. The following null hypotheses were tested:

(1). With students classified as ready by the readiness test, the time of introduction of symbolization will have no effect on students' ability to:

a) interpret addition number sentences.

b) interpret subtraction number sentences.

c) interpret addition and subtraction number sentences.

d) produce addition number sentences.

e) produce subtraction number sentences.

f) produce addition and subtraction number sentences.

g) state answers to addition number sentences.

h) state answers to subtraction number sentences.

i) state answers to addition and subtraction number sentences.

j) interpret, produce, and state answers to addition number sentences.

k) interpret, produce, and state answers to subtraction number sentences.

l) interpret, produce, and state answers to addition and subtraction number sentences.

(2) With students classified as not ready by the readiness test, the time of introduction of symbolization will have no effect on student's ability to:

a) interpret addition number sentences.

b) interpret subtraction number sentences.

c) interpret addition and subtraction number sentences.

d) produce addition number sentences.

e) produce subtraction number sentences.

f) produce addition and subtraction number sentences.

g) state answers to addition number sentences.

h) state answers to subtraction number sentences.

i) state answers to addition and subtraction number sentences.

j) interpret, produce, and state answers to addition number sentences.

k) interpret, produce, and state answers to subtraction number sentences.

l) interpret, produce, and state answers to addition and subtraction number sentences.

SIGNIFICANCE OF THE STUDY

Today, the widespread belief in the importance of readiness is evident in the fact that most first grade children are grouped on the basis of some form of readiness test at the beginning of the school year. Children who are not ready for first grade work would then do essentially kindergarten work for a period of time. The present study

was designed to provide further information in the area of readiness for learning mathematics. In particular, the study will provide information concerning readiness for the meaningful learning of the symbols of mathematics.

The present study also provides direction for replication and parallel studies. The definition of readiness utilized in the present study is easily adaptable to other topics at other grade levels.

Finally, work in the area of reading and mathematics has begun to form a link between concepts in reading or language education and concepts in mathematics education. In the opinion of the investigator the combined efforts of educators in these two fields may have considerable significance in mathematics education. The present study suggests an area of this common interest.

Chapter II

REVIEW AND ANALYSIS OF RELATED LITERATURE

The purpose of this chapter is to describe and analyze the literature related to the present study. The present study was an application in the area of mathematics of a theory concerning readiness for learning the written symbolization of language. In order to apply a theory of readiness for symbolization of language to arithmetic, it was necessary to examine the similarities between language symbols and arithmetic symbols and the learning of each. Hickerson [1] summarized these similarities. Some of the similarities were noted in Chapter I; a more extensive discussion will be presented here.

Hickerson's statements and the rationale of the present study are based on the assumption that written language or written symbols are dependent on oral language or oral symbols. Therefore a discussion of the linguistic theories, a research study, and physiological evidence that support this assumption will be presented in this chapter.

Since the present study was an investigation of readiness, several theories of readiness for mathematics, reading, and language will be discussed. The emphasis, however, will be on the readiness theories applied in reading and language education.

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[1] J. Allen Hickerson, "Similarities Between Teaching Language and Arithmetic," The Arithmetic Teacher, VI (November, 1959), 241.

In the present study, readiness for written symbolization of addition and subtraction was investigated in terms of a child's meaningful learning of the written symbolization of addition and subtraction. It was therefore necessary to precisely define meaning of a written symbol or symbols. The definition of meaning used here was adapted from Fordham.[2] A more extensive discussion of Fordham's definition and other definitions of meaning will be presented in this chapter.

The only study found in which symbolization of arithmetic was delayed for a group of children was by Coxford.[3] A description of Coxford's study and the relationship of Coxford's study to the present study will conclude the review of literature.

SIMILARITIES BETWEEN LANGUAGE AND ARITHMETIC SYMBOLS

Hickerson expressed the main idea behind the present study when he stated:

> Since arithmetic is a system of symbolism, just as language is a system of symbolisms, why shouldn't the accepted principles underlying the understanding and use of language symbols also apply to the understanding and use of arithmetic symbols? It is the writer's conviction that they should apply.[4]

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[2]Dennis L. Fordham, "An Investigation of Third, Fourth, and Fifth Graders' Knowledge of the Meanings of Selected Symbols Associated with Multiplication of Whole Numbers" (unpublished Doctoral dissertation, The University of Georgia, 1974), p. 8.

[3]Arthur Coxford, "The Effect of Two Instructional Approaches on the Learning of Addition and Subtraction Concepts in Grade One" (unpublished Doctoral dissertation, The University of Michigan, 1965).

[4]Hickerson, p. 241.

The present study was an application of some of these principles, those concerned with readiness, to the teaching of addition and subtraction to first grade children.

Hickerson listed the following similarities between language and arithmetic symbols:

Language	Arithmetic
1) Language symbols (words or sentences) represent things, actions, ideas, relationships, etc.	1) Arithmetic symbols (numerals and numbers with operational signs) represent things, actions, ideas, relationships, etc.
2) The meaning of language symbols derive[s] from that which they represent.	2) The meaning of arithmetic symbols derive[s] from that which they represent.[5]

Hickerson implied that arithmetic symbols (numerals, operation and relation signs) form sentences in much the same way that language words are combined to make sentences. The meaning of a language sentence or an arithmetic sentence is dependent on the thing, idea, action or relation that the symbols represent.

Hickerson suggested that the written words or symbols stand for already known spoken words or symbols. Hickerson also implied the dependence of written words or symbols on spoken words or symbols in the following list of similarities between learning language and learning arithmetic:

[5] Hickerson, p. 241.

Learning Language	Learning Arithmetic
3) Listening ~~to~~ spoken word-symbols, singly and in sentences, which represent the things, ideas and events experienced (Learning vocabulary and sentence structure.)	3) Listening to spoken word-symbols, singly and in sentences, which represent the quantitative aspects, quantitative relationships, or quantitative problem-situations found in the things and events experienced. (Learning the vocabulary and sentence structure used in describing things and what is happening to things.)
4) Representing things, ideas, and events through oral language symbols. (Learning to express self and related experiences orally.)	4) Representing quantitative aspects, relationships, and problem situations orally. (Learning to express orally in sentences the quantitative situation, learning to compute orally, and solve problem situations orally.)
5) Identifying written language-symbols and relating them to spoken language-symbols and to first hand experiences. (Learning to read with meaning.)	5) Identifying written arithmetic-symbols and relating them to spoken word-symbols and to first-hand quantitative experiences. (Learning to read arithmetic symbols with meaning.)[6]

Hickerson implied that written symbols are related to spoken symbols when the child is learning to read arithmetic or language symbols with meaning. Evidence to support this implication for language symbols will be presented below. When one examines the similarities between language and arithmetic symbols, the implication also seems plausible for arithmetic symbols.

[6]Hickerson, pp. 242-243.

DEPENDENCY OF WRITTEN LANGUAGE SYMBOLS ON ORAL LANGUAGE SYMBOLS

The linguist Hill stated, "The final fact is that all writing systems are essentially representations of the forms of speech, rather than representatives of ideas or objects in the nonlinguistic world."[7] According to Hill, linguists and anthropologists assume that if the remains of any past community show signs of social organization, the community must have had some form of language. Artifacts of such communities have been found that are much older than the remains of any communities that had writing.

The theory that oral language is primary, and that written language is secondary and dependent on oral langauge, is also supported by Fries, a linguist whose special field is the historical and descriptive study of the English language. Fries states, "Language must come first. . . . In comparison with the tremendously ancient activity of talking, the processes of writing and reading are much later inventions."[8] Fries also states that "An understanding of the nature and functioning of language must form the foundation upon which to build an understanding of the derived processes of writing and reading."[9] A

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[7]Archibald A. Hill, Introduction to Linguistic Structures, From Sound to Sentence in English (New York: Harcourt, Brace & World, Inc., 1958), p. 2.

[8]Charles C. Fries, Linguistics and Reading (New York: Holt, Rinehart and Winston, Inc., 1963), pp. 113-114.

[9]Fries, Linguistics and Reading, p. 113.

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discussion of the literature in reading education, which tends to support this statement, will be presented later.

Tatham investigated whether children read better when the reading material is closely related to the children's own language. A previous study on oral language patterns of second and fourth grade children was used to determine "frequent" and "infrequent" oral language patterns. Two reading comprehension tests were developed. Test A was composed of frequent oral language patterns and Test B was composed of infrequent oral language patterns. All children took both tests but some children took Test A first while others took Test B first. The order of testing was determined by random assignment. Tatham found that "a significant number of second and fourth graders comprehend material written with frequent oral language patterns better than material written with infrequent oral language patterns."[10]

The main conclusion of Tatham's study was that beginning reading materials should be written to more closely approximate the structure of a child's speech. Her study also implied the dependency of reading on spoken language; children had better comprehension of reading material which more closely approximated their own speech.

There is also physiological evidence supporting the theory that written language is dependent on oral language. Most people engage in silent or implicit speech when they are reading. Silent or implicit speech occurs during reading when minute movements occur in the muscles

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[10] Susan Masland Tatham, "Reading Comprehension of Materials Written with Selected Oral Language Patterns: A Study at Grades Two and Four," Reading Research Quarterly, V, (Spring, 1970), 423.

that would move if the same words were whispered or said aloud. Deaf people produce the same movements in their hands while reading.

Davies reviewed and summarized the research from 1868 to 1970 that pertained to implicit speech and reading. He concluded that prior to 1960,

> the accumulated opinions of specialists in the field of reading support the theory that implicit speech may aid (reading) comprehension in the primary grades but that it can be a deterrent to adequate rate in the intermediate and upper grades.[11]

However, in 1960 Edfelt reported the results of his experiments with implicit speech and concluded that "silent speech is universal during silent reading; it increases with the difficulty of the material; efforts to eliminate it should be discontinued."[12] The review by Davies of the implicit speech research from 1960 to 1970 indicated that most of the researchers during this period agreed with Edfelt. In his summary of the research from that period, Davies confirmed the fact that

> implicit speech is a normal adjunct to the reading process; that it is a natural developmental comprehension reinforcer; and that, consequently, classroom techniques for its repression should be minimized, if not abandoned altogether.[13]

The fact that implicit speech accompanies silent reading and that most researchers think silent speech actually aids reading

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[11]William C. Davies, "Implicit Speech - Some Conclusions Drawn from Research," Some Persistent Questions on Beginning Reading, ed. Robert Aukerman (Newark: International Association, Inc., 1972) p. 173.

[12]Ake W. Edfelt, Silent Speech and Silent Reading, (Chicago: University of Chicago Press, 1960), p. 154.

[13]Davies, p. 174.

comprehension is further evidence that comprehension of written language is dependent on spoken language.

READINESS

In mathematics education, there seem to be three main theories concerning readiness. Some educators subscribe to the theory that readiness is a function of the cognitive development of the child. This view of readiness was summarized by Shulman:

> To identify whether the child is ready to learn a particular concept or principle, one analyzes the structure of that to be taught and compares it with what is already known about the cognitive structure of the child at that age. If they are consonant, it can be taught; if they are dissonant, it cannot.[14]

A second theory, that of Bruner, also considers the cognitive development of the child but in addition includes the child's readiness for different levels of the subject matter. Bruner's statement, ". . . any subject can be taught effectively in some intellectually honest form to any child at any stage of development",[15] characterizes this theory. Bruner's position is that one need not wait for the child to reach a certain readiness state, but one may modify the material to be taught to conform to the child's immediate readiness state. Bruner further clarified his position when he said,

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[14]Lee S. Shulman, "Psychological Controversies in the Teaching of Science and Mathematics," The Science Teacher (September, 1968), p. 36.

[15]Jerome S. Bruner, The Process of Education (New York: Vintage Books, 1963), p. 33.

"Readiness, I would argue, is a function not so much of maturation - which is not to say that maturation is not important - but rather of our intentions and our skill at translation of ideas into the language and concepts of the age we are teaching."[16]

A third theory, and the theory of readiness to which the present study is most closely related, is that readiness is a function of prerequisite learnings. Gagne described this theory in the following statement:

Each learner approaches each new learning task with a different collection of previously learned prerequisite skills. To be effective, therefore, a learning program for each child must take fully into account what he knows how to do already and what he doesn't know how to do already."[17]

In reading and language education, readiness is often viewed as a function of prerequisite learnings. Downing and Thackray stated that

. . . readiness does not necessarily imply that a child achieves this state only through growth or maturation. He may also arrive at readiness through having completed the prior learnings on which the new learnings will be based.[18]

One of the more important "prerequisite learnings" necessary for beginning reading that Downing and Thackray describe is a well developed oral vocabulary.

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[16]Jerome S. Bruner, "On Learning Mathematics," The Mathematics Teacher (December, 1960), p. 610.

[17]Robert M. Gagne, "Some New Views of Learning and Instruction," Phi Delta Kappan (May, 1970), p. 468.

[18]John Downing and D. V. Thackray, Reading Readiness (London: University of London Press Ltd., 1972), p. 9.

Other educators in reading and language education consider a child's oral vocabulary or language facility to be an important prerequisite for learning to read. Henceforth, when the terms "language facility" or "verbal facility" are used they will refer to facility with spoken language rather then written language. According to Bond, there is also an advantage in that "language facility is one of the more important readiness factors that are definitely trainable."[19]

Language facility is also considered to be an important reading readiness factor in the approach to the teaching of reading, learning to read through experience. This approach is based on the argument that meaning and understanding in reading must have their basis in the experience of the child. A child beginning to learn to read through experience will often have the opportunity to build his own reading materials; thus the child will be reading about his own experiences in his own language. A different approach is also taken toward teaching phonetics. A child is taught to symbolize his own speech sounds rather than assign a sound to a symbol or symbols.

Lee and Allen list the following as some of the basic assumptions behind the approach to learning to read through experience:

> Reading is concerned with words that arouse meaningful responses based on the <u>individual experience</u> of the learner.
>
> Words have no inherent meaning.
>
> Spoken words are sound symbols which arouse meaning in the mind of the listener.

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[19] Guy L. Bond and others, Pre-Primers, <u>Three of Us, Play With Us, Fun with Us</u>, with Teacher's Guide (Chicago: Lyons and Carnahan, 1954), p. 18.

> Written words are visual symbols, which when associated with known sound symbols, arouse meaning in the mind of the reader.[20]

Lee and Allen implied that it is the spoken words that arouse meaning. The written words must first be associated with spoken words, and the spoken words then arouse meaning. Thus an adequate spoken vocabulary is an essential prerequisite for a child to learn to read through experience.

Tinker and McCullough also emphasize the importance of spoken words in beginning reading. Tinker and McCullough stated:

> For the beginner, learning to read entails learning that printed symbols stand for speech. The child reads when he says the correct printed words and recognizes their meaning because of his previous experience in comprehending speech in meaningful sequence. He discovers that printed words "talk" sense.[21]

It is possible that the printed symbols of mathematics as well as language may lack meaning to many children because the children lack the adequate verbal facility necessary to make the printed symbols "talk sense." According to Tinker and McCullough, "Only when the printed symbols stand for words used meaningfully in his own speech is the child ready to read successfully."[22]

Harris defines beginning reading in much the same way as Tinker and McCullough. Harris stated:

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[20] Dorris M. Lee and R. V. Allen, Learning to Read Through Experience (New York: Appleton - Century - Crofts, 1963), p. 2.

[21] Miles A. Tinker and Constance M. McCullough, Teaching Elementary Reading (New Jersey: Prentice-Hall Inc., 1975), p. 7.

[22] Tinker and McCullough, pp. 81-82.

> We may define reading as the act of responding with appropriate meaning to printed or written verbal symbols. For the beginner, reading is mainly concerned with learning to recognize the printed symbols which represent speech and to respond intellectually and emotionally as he would if the material were spoken rather than printed.[23]

It seems, therefore, that many reading experts believe successful reading is dependent on the child's ability to relate the written symbols to spoken symbols. The spoken symbols provide a link between written symbols and the meaning to be assigned to the written symbols. When this link is broken, i.e., the child does not have adequate verbal facility, the child cannot assign meaning to the written symbols. For this reason, verbal facility is an important readiness factor for learning to read. When one also considers the similarities between language and arithmetic symbols and the learning of each, it seems plausible that verbal facility in saying the words represented by arithmetic symbols might be a link between the written symbols and ~~their~~ meaning. Therefore, verbal facility with arithmetic symbols is plausible as a readiness factor for learning to read and work with arithmetic symbols.

Some people may argue that arithmetic symbols are a form of language but are a foreign language. In learning a foreign language, verbal facility with the language is also important. Fries considers verbal facility to be important in the learning of any new language. Fries states:

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[23]Albert J. Harris, How To Increase Reading Ability (New York: Davis McKay Company, Inc., 1970), p. 3.

> No matter if the final result desired is only to read the foreign language the mastery of the fundamentals of the language must be through speech. The speech is the language. The written record is but a secondary representation of the language. To "master" a language it is not necessary to read it, but it is extremely doubtful whether one can really read the language without first mastering it orally.[24]

In the present study, arithmetic symbols will be viewed as part of a first language, not a foreign language. However, it is important to note that even if arithmetic symbols are viewed as a foreign language, verbal facility is still a link between the printed symbols and their meaning.

MEANING

Since the purpose of the present study was to investigate whether the attainment of the previously defined readiness state affects the meaningful learning of the symbolization of addition and subtraction, it was necessary to first define meaning of a symbol. The term "meaning" as applied to mathematics has been closely linked to the writings of Brownell. According to Brownell, meanings ". . . must be sought in the mathematical relationships of the subject itself, in its concepts, generalizations, and principles."[25]

A second definition of meaning, or meaningful learning, is that of Ausubel who states, "Meaningful learning takes place if the learning

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[24]Charles C. Fries, Teaching and Learning English as a Foreign Language (Ann Arbor: University of Michigan Press, 1945), p. 6.

[25]William A. Brownell, "The Place of Meaning in the Teaching of Arithmetic," Elementary School Journal, XLVII (January, 1947), 257.

is related in a nonarbitrary and nonverbatim fashion to the learner's existing structure of knowledge."[26] Neither Brownell's nor Ausubel's definitions are sufficiently precise to be used to assess meaning.

A more precise definition of meaning was stated by Van Engen:

> In any meaningful situation there are always three elements. (1) There is an event, an object or an action. In general terms there is a referent. (2) There is a symbol for the referent. (3) There is an individual to interpret the symbol as somehow referring to the referent. Thus in an arithmetic situation, the phrase "½ an apple" is the symbol. The referent is the half apple, and the interpretation, if meaningful, is the act of cutting the apple. It is important to remember that the symbol always refers to something outside itself.[27]

Thus a symbol is meaningful to a child if the child can interpret the symbol as referring to an object or an action on an object. It is possible to assess whether or not a child can interpret a given symbol.

Fordham's definition of meaning of a symbol also implied the existence of a referent and an individual to interpret the symbol as referring to the referent or an action on the referent. According to Fordham,

> A meaning of the symbol a x b is a pairing in the mind of an individual, of the symbol with some action that is appropriate for the symbol. The appropriate actions are those that provide an answer to the question, "How many are represented by the symbol?"[28]

[26] David P. Ausubel, "Facilitating Meaningful Verbal Learning in the Classroom," Teaching Mathematics: Psychological Foundations, eds. Joe Crosswhite and others (Worthington, Ohio: Charles A. Jones Publishing Company, 1973), p. 150.

[27] Henry Van Engen, "An Analysis of Meaning in Arithmetic I," Elementary School Journal, XLIX (February, 1949), 323.

[28] Fordham, p. 8.

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The definition of meaning of the symbols "a + b = c" and "a - b = c" used in the present study was adapted from Fordham's definition. It should be noted that Fordham's definition and the definition used in the present study were defined operationally, thus providing a basis for measuring meaning or lack of meaning associated with a symbol by a child. Fordham stated that "in order to demonstrate that he knows a meaning of a symbol, an individual must either carry out or describe some action that is appropriate for the symbol."[29] The child is thus demonstrating that the "pairing in his mind" of the symbol with the referent exists by interpreting the symbol. In the present study, a child could also demonstrate that the same "pairing in his mind" exists by producing the appropriate symbol when given a referent or action on the referent.

THE COXFORD STUDY

Description of the Study

In 1965, Coxford[30] conducted a study in which he taught six classes of first grade children for the full school year. The main purposes of his study were to investigate the effect of two instructional approaches to teaching subtraction and the effect of immediate versus delayed symbolization on first grade children's arithmetic achievement. Since only the investigation of the time of symbolization

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[29] Fordham, p. 8.

[30] Arthur Coxford, "The Effect of Two Instructional Approaches on the Learning of Addition and Subtraction Concepts in Grade One" (unpublished Doctoral dissertation, The University of Michigan, 1965).

is directly related to the present study, the instructional approaches will be described only as Treatment 1 and Treatment 2, and the results will not be discussed.

Six classes in three elementary schools participated in the study. Previous achievement data showed that four classes were high ability and two were low ability. Separate analyses were done for the low and high ability classes.

The four high ability classes were assigned to one of the two treatments and were designated as an early or late symbolization group. The low ability classes were assigned to one of the two treatments; one was designated an early and the other a late symbolization group. The assignment of treatments and time symbolization of the six classes are illustrated in Figure 3.

Time of Symbolization	High Ability		Low Ability
	Treatment 1	Treatment 2	Treatment 1
Early	Class 1	Class 2	Class 5
Late	Class 3	Class 4	Class 6

Figure 3

Diagram of Assignment of Classes to Treatments
In Coxford's Study

In the early symbolization classes, written symbolization of addition and subtraction was introduced simultaneously with the concepts. In the late symbolization classes, written symbolization was delayed for six weeks. During this time, the classes used the phrase "number sentence" but did not use words such as "plus," "minus " or "equals." Also during this six week delay, the late symbolization groups were exposed to written number sentences of the form "A and B is C" and "C take away B is A." In the present study, the delayed groups of children also used the phrase "number sentence," but used the words "plus," "minus" and "equals," and were not exposed to written number sentences of any form.

With the high ability classes, no significant differences in mean arithmetic achievement were found between the early and late symbolization groups except in the area of problem solving. The early symbolization classes scored significantly higher on problem solving than the late symbolization classes.[31]

With the low ability classes, no statistically significant differences were found in mean arithmetic achievement. However, the children in the late symbolization class had consistently greater achievement scores in subtraction application and transfer than did the children in the early symbolization class. The mean differences were statistically reliable when individual differences were controlled by the results of the Lorge-Thorndike Intelligence Test.[32]

Coxford also stated that "observations made throughout the year in the lower ability late symbolization class and in the TA [Treatment 2]

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[31]Coxford, p. 98. [32]Coxford, p. 92.

late symbolization class suggested that the delaying symbolization facilitated immediate learning."[33]

Relationship to the Present Study

Coxford recommended that ". . . the effects of symbolization should be studied further because of immediate difficulties with the use of mathematical symbolization and because more symbolization is entering elementary school mathematics curricula."[34] The present study implements that recommendation, utilizing a precise definition of readiness for symbolization.

The main difference between Coxford's study and the present study is the consideration of a child's readiness for symbolization. The delay of symbolization in Coxford's study was similar to the treatment given the ready children in the delayed symbolization group of the present study. That is, symbolization was delayed for a predetermined number of weeks. However, in the present study, the not ready children in the delayed symbolization group had symbolization delayed until the children were determined to be ready. Thus the time of delay varied from child to child.

It is possible that many of the high ability children in Coxford's study were ready for symbolization according to the definition of readiness used in the present study. This could have accounted for the higher performance of the early symbolization classes especially in problem solving. It is also possible that many of the lower ability children in Coxford's study were not ready for symbolization. This

[33]Coxford, p. 103. [34]Coxford, p. 103.

could have accounted for the higher performance of the late symbolization class. It should be noted, however, that symbolization was delayed for the lower ability children for a predetermined six week period. A child's readiness for symbolization was not considered in Coxford's study.

The analogy between learning to read language symbols and arithmetic symbols, linguistic considerations, physiological evidence, opinions of experts in reading, and the results of a research study lead to the conclusion that a child's verbal facility with mathematical symbols may be an important prerequisite for the child's meaningful learning of the symbols. This study was an attempt to explore that area. In Chapter III, the experimental procedures and statistical analyses used in the study will be discussed.

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Chapter III

SUBJECTS, INSTRUMENTS, AND RESEARCH DESIGN

The first purpose of this chapter is to describe the subjects who participated in the study and the curriculum of the subjects' school. The second purpose is to describe the testing instruments and lesson plans used in the study. The final purpose is to describe the research design, i.e., the selection of subjects, the assignment of subjects to treatments, the treatments, the gathering of data for investigation of the hypotheses, and the statistical methods used to analyze the data.

THE SUBJECTS AND THE SUBJECTS' SCHOOL CURRICULUM

The Subjects

The subjects were 38 first grade students in David C. Barrow Elementary School in Athens, Georgia. When the study began in September, 1975, the subjects ranged in age from five years nine months to six years ten months. Of the subjects, 18 were female and 20 were male.

Deviation Intelligence Quotient (DIQ) scores from the Primary II level of the Otis-Lennon Mental Ability Test were available for all subjects. These scores have a mean of 100 and a standard deviation of 16 points. The mean DIQ score of the subjects was 109.66 with a standard deviation of 15.74. The DIQ scores of the subjects ranged from 80 to 135.

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The Curriculum of the Subjects' School

During the school year preceding the present study (1974-1975), the first grade mathematics curriculum of Barrow School was based on the objectives of the Individually Prescribed Instruction (IPI) mathematics program. The IPI instructional materials were not used, but the teachers designed and shared materials for their classes using the IPI objectives as a guideline.

The IPI objectives are divided into seven levels, labeled from lowest to highest A, B, C, D, E, F, and G. First grade students making average progress were expected to have mastered the Level A objectives by the end of the school year. These objectives are included in Appendix E.

The Level A objectives were partitioned into the following sections:

(1) Numeration and Place Value

(2) Addition and Subtraction

(3) Fractions

(4) Money

(5) Time

At a meeting with the first grade teachers in August, 1975, it was agreed that the IPI objectives and teacher's instructional materials would be used for the first topic, Numeration and Place Value. The IPI objectives for this topic were limited to the digits zero through nine. It was agreed that when the teachers decided most of the first grade children had mastered the IPI objectives for this topic, the present study would begin. The teachers also agreed to use the instructional

materials and objectives prepared for the present study for all students in the first grade.

TESTING INSTRUMENTS AND LESSON PLANS

The Readiness Test

As noted in Chapter I, a child was defined to be ready for the introduction of symbolization of addition and subtraction when he had mastered the following objectives verbally, perhaps with the aid of pictures or manipulatives:

(1) Given objects or a picture that illustrate the union of two sets, the student states the sum and says the number sentence.

(2) Given a collection of objects, and given verbally an addition number sentence $a + b = c$, where a, b, and c are small whole numbers less than 10, the student forms sets having a and b elements respectively, illustrates the union of the two sets, and states the sum.

(3) Given objects or a picture that illustrate the removal or partitioning of a subset, the student states the difference and says the number sentence.

(4) Given a collection of objects and given verbally a s[illegible]action number sentence, $a - b = c$, where a, b, and c are small whole numbers less than 10 and $b < a$, the student forms a set having a elements and removes or partitions a subset having b elements and states the difference.

A readiness test, based on these four objectives, was constructed by the investigator. This test was used in the initial stages of the study to classify subjects as "ready" or "not ready" for written symbolization of addition and subtraction. A child was classified as

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"ready" if the child demonstrated mastery of all items on the readiness test. A child was otherwise classified as "not ready."

During the month of September, 1975, the first grade children at Barrow School were administered two tests as part of the Project for the Mathematical Development of Children (PMDC). These tests included the Key Math Test,[1] an individually administered mathematical diagnostic test, and the PMDC Test,[2] an individually administered test prepared by PMDC. To avoid having to disrupt the school schedule with a third long individually administered test, selected items from the Key Math Test and the PMDC Test were used as part of the readiness test. Nine items were selected from the Key Math Test and four items were selected from the PMDC Test that were judged to partially assess subjects' mastery of objectives one and two of the definition of readiness given in Chapter I. The nine items from the Key Math Test required a subject to state a sum or difference when given a picture stimulus accompanied by a story. The four items from the PMDC Test required a subject to state a sum or difference when given a story stimulus.

A brief interview was designed by the investigator to further assess subjects' mastery of objectives one and two and to assess subjects' mastery of objectives three and four of the definition of readiness. Two items required a subject to say number sentences and

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[1]Austin J. Connolly, William Nachtman, and E. Milo Pritchett, Key Math Diagnostic Arithmetic Test (Circle Pines: American Guidance Service, Inc., 1971).

[2]Project for the Mathematical Development of Children, "Mathematics Test: Grade 1", PMDC, Florida State University, Tallahassee, Florida, unpublished, September, 1975.

state sums or differences when given a picture stimulus (Items I and II). Four items required a subject to manipulate objects to interpret a given number sentence (Items IIIa, IIIb, IIIc, and IIId). Two items required a subject to state a sum or difference when given a picture stimulus (Items IV and V).

The total readiness test included the selected items from the Key Math Test, the PMDC Test and the interview prepared by the investigator. The objectives of the definition of readiness and the readiness test items are included in Appendix A. A child was classified as ready for the introduction of symbolization of addition and subtraction if he correctly answered all items on the readiness test.

The Posttest

Since the purpose of the present study was to investigate the relationship between children's readiness for written symbolization and their meaningful learning of the written symbolization, it was necessary to develop a means of assessing the meaning assigned to written symbols by the subjects.

As noted in Chapter I, an individual may demonstrate that he knows the meaning of a symbol by one of the following two ways: (1) Given a symbol, the individual must interpret the symbol by describing or carrying out an action that is appropriate for the symbol. (2) Given an action or description of an action that is appropriate for a symbol, the individual must produce the symbol.

When the symbol is an addition or subtraction number sentence, it is also necessary that the individual state the correct sum or difference. It is assumed that the child who can do all three of the

above, i.e., interpret, produce, and state the answer, for a given symbolic number sentence, demonstrates that the symbols are more meaningful to him than they are to a child who can do only one or two of the above. Therefore, to measure meaningfulness, a posttest was designed to assess a subject's proficiency in three areas: production of number sentences, interpretation of number sentences, and statement of answers to number sentences.

All possible combinations of stimuli and responses were considered in the construction of the posttest; however, some combinations were omitted to reduce the length of the test. The combinations of stimuli and response included on the posttest are illustrated in Table 1.

Items were included in the posttest requiring subjects to interpret given number sentences, produce number sentences, and state answers to number sentences. The item numbers and number of items requiring subjects to interpret, produce, and state answers to number sentences are presented in Table 2, Table 3, and Table 4 respectively. The posttest is included in Appendix D.

The Lesson Plans

A unit of lesson plans and activities was developed for the study by the investigator. The unit consisted of 63 activities based on the Level A IPI objectives for addition and subtraction. The nine objectives for the unit, the corresponding IPI objectives and the activity numbers for each objective are included in Appendix B.

Each activity was divided into the following sections:

(1) Concrete-pictorial (CP): Lessons in the concrete-pictorial sections involved the use of concrete objects or pictures. The lessons

Table 1

Combinations of Stimuli and Responses on the Posttest

Production of Answers

Possible Stimuli	Possible Response	Combinations Included		Combinations Omitted	
(1) verbal number sentence	(a) verbal	1-a	2-a	1-b	3-b
(2) written number sentence	(b) written	2-b	3-a		
(3) story problem					

Production of Number Sentences

Possible Stimuli	Possible Response	Combinations Included		Combinations Omitted
(1) objects	(a) verbal	1-a	1-b	none
(2) pictures	(b) written	2-a	2-b	
(3) story problem		3-a	3-b	

Interpretation of Number Sentences

Possible Stimuli	Possible Response	Combinations Included		Combinations Omitted
(1) verbal number sentence	(a) picture	1-a	1-b	2-a
(2) written number sentence	(b) objects	2-b		

Table 2

Item Numbers and Number of Items on Posttest Requiring Interpretation of Verbal and Written Number Sentences

Addition:

		Number Sentence: Verbal	Number Sentence: Written
Interpretation	Objects	VIII-1-b VIII-2-b V-1-b V-2-b (4)	VII-a (1)
	Pictures	(0)	VI-a VI-b (2)

Subtraction:

		Number Sentence: Verbal	Number Sentence: Written
Interpretation	Objects	VII-3-b V-3-b V-4-b (3)	VII-b (1)
	Pictures	(0)	VI-c VI-d (2)

Total:

		Number Sentence: Verbal	Number Sentence: Written
Interpretation	Objects	V-1-b VIII-1-b V-2-b VIII-2-b V-3-b VIII-3-b V-4-b (7)	VII-a VII-b (2)
	Pictures	(0)	VI-a VI-b VI-c VI-d (4)

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Table 3

Item Numbers and Number of Items on Posttest Requiring Production of Verbal and Written Number Sentences

Addition:

Number Sentence	Picture	Story Problem	Objects
Verbal	II-1 II-2 (2)	III-1 (1)	IX-1 (1)
Written	IV-1 IV-2 (2)	III-2 (1)	IX-4 (1)

Subtraction:

Number Sentence	Picture	Story Problem	Objects
Verbal	II-3 II-4 (2)	III-3 (1)	IX-3 (1)
Written	IV-3 IV-4 (2)	III-4 (1)	IX-2 (1)

Total

Number Sentence	Picture	Story Problem	Objects
Verbal	II-1 II-4 II-2 II-3 (4)	III-1 III-3 (2)	IX-1 IX-3 (2)
Written	IV-1 IV-4 IV-2 IV-3 (4)	III-2 III-4 (2)	IX-2 IX-4 (2)

Table 4

Item Numbers and Number of Items on Posttest Requiring an Answer to Verbal and Written Number Sentences

Addition:

		Stimuli: Verbal Number Sentence	Stimuli: Written Number Sentence	Stimuli: Story Problem
Response	Verbal	I-a I-b I-c I-d (4)	VIII-1-a VIII-2-a (2)	III-1 III-2 (2)
	Written	(0)	V-1-a V-2-a (2)	(0)

Subtraction:

		Stimuli: Verbal Number Sentence	Stimuli: Written Number Sentence	Stimuli: Story Problem
Response	Verbal	I-e I-f I-g I-h (4)	VIII-3-a (1)	III-3 III-4 (2)
	Written	(0)	V-3-a V-4-a (2)	(0)

Total:

		Stimuli: Verbal Number Sentence	Stimuli: Written Number Sentence	Stimuli: Story Problem
Response	Verbal	I-a I-e I-b I-f I-c I-g I-d I-h (8)	VIII-1-a VIII-2-a VIII-3-a (3)	III-1 III-2 III-3 III-4 (4)
	Written	(0)	V-1-a V-2-a V-3-a V-4-a (4)	(0)

were designed to be conducted in a verbal mode with emphasis on verbalization on the part of the teacher and the students as a group. Written symbols were not included in these lessons.

(2) Verbal (V): Lessons in the verbal sections involved oral responses primarily on the part of the students. The lessons were designed to encourage the students to say number sentences when given actions or pictures illustrating the number sentences, and to encourage students to interpret given number sentences by describing or demonstrating actions appropriate to the number sentences.

(3) Symbolic (S): Lessons in the symbolic sections involved written symbols. These lessons were designed to be similar to pages typical of first grade arithmetic texts.

Many of the verbal (V) lesson plans were marked with the phrase, "Mastery indicative or readiness obj. ______." The appropriate objective of the unit was inserted in the blank. If a subject mastered one of these activities for a particular unit objective on two consecutive days, he was considered to be ready for the introduction of the symbolization for that objective. Sample activities from the lesson plans, called the teacher's manual, and sample worksheets are included in Appendix C.

THE RESEARCH DESIGN

The Selection of Subjects

During the month of September, 1975, the investigator administered the interview section of the readiness test to 66 first grade children at Barrow School. Scores on the selected Key Math Test and

PMDC Test items were also obtained for the 66 subjects. These items were combined with the interview items to form the 21 item readiness test. Of the 66 subjects tested, 17 subjects successfully answered all 21 items. These 17 subjects were classified as ready for written symbolization and the remaining 49 children were classified as not ready for written symbolization.

The Assignment of Subjects to Treatments

Key Math Test total raw scores were obtained for the 66 subjects. The ready children were paired on the basis of these scores. Except for one pair, the Key Math Test total raw scores of each pair did not differ by more than 6.6 points, i.e., two times the standard error of measurement for the scores according to the Key Math Test manual. This pairing procedure resulted in eight pairs of ready children. One member of each pair was randomly assigned to a delayed symbolization group (RD), and the other member was assigned to an immediate symbolization group (RI). The Key Math Test raw score of each subject in each group and the mean Key Math Test total raw score for each group are reported in Table 5.

The not ready subjects were paired on the basis of the readiness test scores and Key Math Test total raw scores. Between each pair of scores, a difference of one point was allowed on the readiness test scores and again a difference of 6.6 points was allowed between the Key Math Test scores. This pairing procedure resulted in 11 pairs of not ready subjects. The remaining not ready subjects could not be matched. One member from each pair was randomly assigned to a delayed symbolization group (ND) and the other member was assigned to an immediate

Table 5

Ready Subjects
Key Math Test Total Raw Score of Each Subject
and Mean Key Math Test Total Raw
Score for Each Group

Ready Immediate Group (RI)		Ready Delayed Group (RD)	
Subject	Total Raw Score	Subject	Total Raw Score
RI-1	80	RD-1	74
RI-2	75	RD-2	69
RI-3	71	RD-3	66
RI-4	59	RD-4	65
RI-5	55	RD-5	63
RI-6	52	RD-6	54
RI-7	49	RD-7	52
RI-8	48	RD-8	52
Mean	61.12	Mean	61.87

symbolization group (NI). The readiness test score and Key Math Test total raw score for each subject in each group and the mean readiness test score and Key Math Test total raw score of each group are reported in Table 6.

The overall method of selection of subjects and assignments of subjects to treatments is diagrammed in Figure 4. The selection and assignment of subjects was accomplished in September, 1975. During this time, the subjects were working on the IPI objectives for numeration and place value under the direction of their regular teachers.

The Treatments

At the time of the present study, Barrow School had four regular first grade teachers, two aides, and two interns from the University of Georgia elementary education program. All of these people agreed to participate in the instructional part of the study. In addition, the investigator and a doctoral student in mathematics education at the University of Georgia also participated in the instructional part of the study. The teacher aides were assigned groups of students who were not in the study and who needed remedial work with numeration. Thus the eight instructors in the study included the four first grade teachers, two interns, one doctoral student, and the investigator. Each of the four treatment groups was divided between two of the eight instructors.

There were four treatment groups in the study: the Ready Delayed Symbolization group (RD), the Ready Immediate Symbolization group (RI), the Not Ready Delayed Symbolization group (ND), and the Not Ready Immediate Symbolization group (NI). Additional first grade children who were not included in the study were assigned to the various groups

Table 6

Not Ready Subjects
Key Math Test Total Raw Score and Readiness Test
Score of Each Subject and Mean
Scores of Each Group

Not Ready Immediate Group (NI)			Not Ready Delayed Group (ND)		
Subject	Readiness Test Score	Key Math Test Score	Subject	Readiness Test Score	Key Math Test Score
NI-1	17	57	ND-1	17	57
NI-2	17	49	ND-2	17	53
NI-3	16	40	ND-3	15	41
NI-4	15	39	ND-4	15	40
NI-5	14	42	ND-5	14	43
NI-6	12	31	ND-6	12	30
NI-7	7	27	ND-7	7	27
NI-8	7	29	ND-8	7	29
NI-9	6	34	ND-9	6	31
NI-10	5	33	ND-10	5	35
NI-11	4	31	ND-11	4	31
Means	10.90	37.45	Means	10.82	37.91

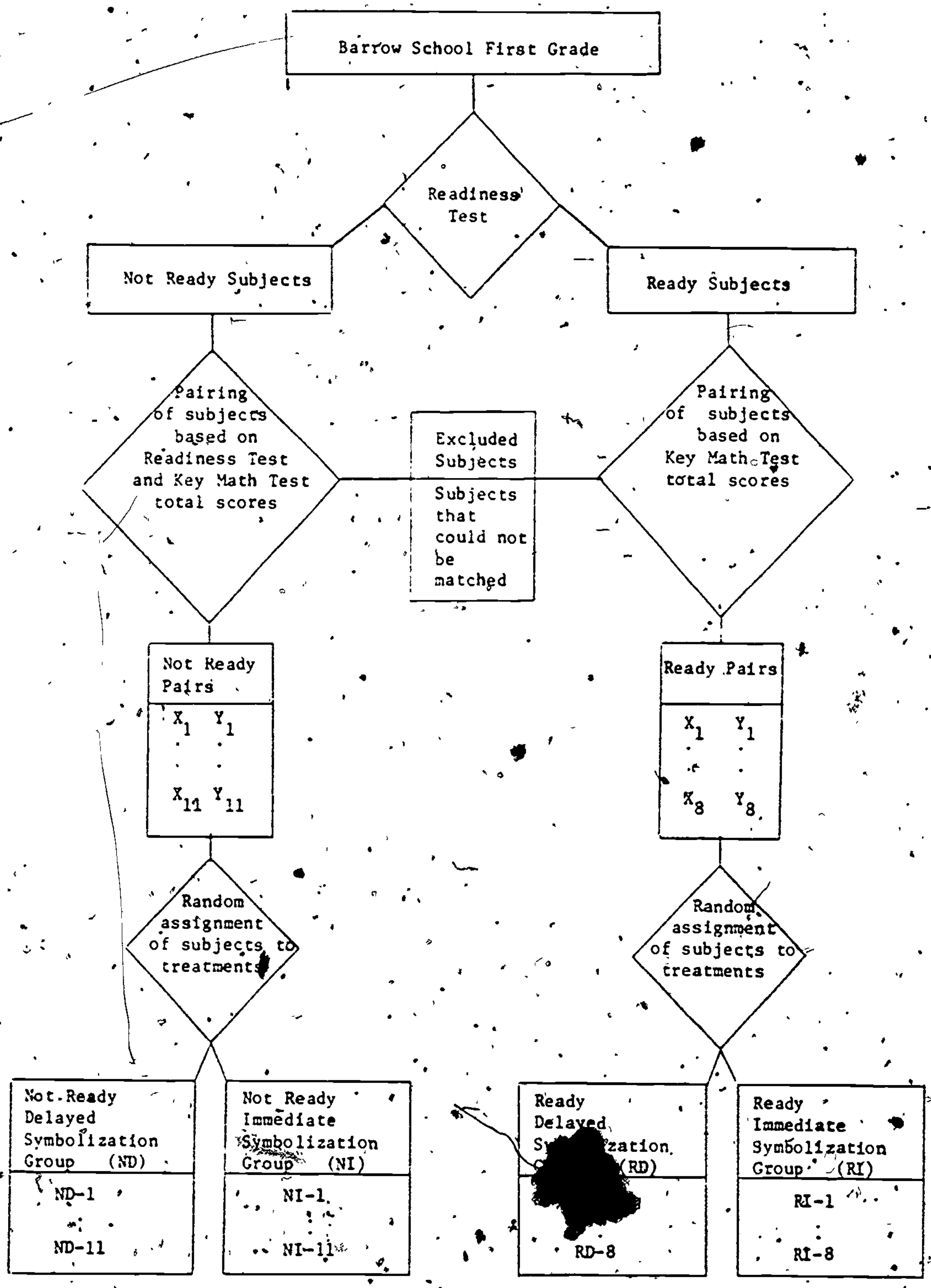

Figure 4

Diagram of the Methods of Selection of Subjects and Assignment of Subjects to Treatment Conditions

by the teachers. The instructors of all treatment groups used the unit on addition and subtraction prepared for the study, but the time of introduction of symbolization was varied for the different groups.

Each of the four treatments was conducted simultaneously for a 12 week period. The RI and NI groups were combined for instructional purposes; these two groups were taught by four regular teachers at Barrow School. The subjects in these groups experienced a treatment in which written symbolization of addition and subtraction was introduced simultaneously with the concepts. These groups were therefore similar to control groups since, in the typical first grade curriculum, a concept and its symbolization are usually introduced together. The subjects in the RI and NI groups worked all three sections of each activity, the concrete-pictorial, the verbal and the symbolic sections.

The subjects in the RD group were divided between the two interns for instructional purposes. The subjects in this group experienced a treatment in which written symbolization was delayed for five weeks. It was originally planned that the symbolization be delayed for six weeks, but observations during the fifth week indicated that the subjects were becoming bored and frustrated. These subjects worked only the concrete-pictorial and verbal sections of each activity during the five week delay period. At the end of the five week delay, the subjects worked the previously skipped symbolic sections of each activity. For the remainder of the study, these subjects worked all three sections of each activity.

The subjects in the ND group were divided between the investigator and a doctoral student for instructional purposes. The subjects in this group experienced a treatment in which written symbolization

was delayed for each subject until the subject was judged to be ready. A subject was judged to be ready for the symbolization of each of the nine objectives of the instructional unit when he had demonstrated mastery, on two consecutive days, of a verbal section on an objective that was marked with the phrase, "Mastery indicative of readiness obj. ____." Ideally, once a subject demonstrated mastery of these sections, the written symbolization of the objective would then be introduced to the subject. However, for practical reasons, this introduction was sometimes delayed for a few days until several other subjects were judged to be ready. During the delay period, these subjects worked only the CP and V sections of each activity. When a subject had mastered an objective verbally, he then worked the previously skipped S sections of activities for that objective. The subject then proceeded to the activities for the next objective, but again worked only the CP and V sections until the objective was mastered verbally.

The Data and the Statistical Methods

After the twelfth week, the posttest was administered to all subjects. Scores on 12 sections of the posttest were recorded for each subject. Each score was related to one of the 12 null hypotheses listed in Chapter I. For example, the score related to hypothesis 1-a or 2-a was the number of addition number sentences correctly interpreted. Each null hypothesis for both ready and not ready subjects was tested by

the Wilcoxon Signed Ranks Test procedure described by Conover.[1] The hypotheses were tested at the level of significance $\alpha = .05$.

Confidence intervals were also constructed for each of the twelve null hypotheses for both ready and not ready subjects. The 90% and 95% confidence intervals were constructed by a geometric construction procedure described by Conover.[2]

Although the activities for the unit included some activities on missing addend number sentences, i.e., number sentences of the form a + ___ = c or a - ___ = c; none of the subjects in the present study had been introduced to these activities at the time of the posttest. The posttest contained six items that required work with missing addend number sentences, items I-i, I-j, III-5-a, III-5-b, IV-6-a, and IV-6-b. The data for these items were analyzed separately from the data for the other items in the posttest. The Wilcoxon Matched Pairs Signed Ranks procedure described by Conover[3] was used to test the null hypotheses:

3-a: The time of introduction of symbolization will have no effect on ready subjects' ability to state answers to and produce missing addend number sentences.

3-b: The time of introduction of symbolization will have no effect on not ready subjects' ability to state answers to and produce missing addend number sentences.

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[1] W. J. Conover, Practical Nonparametric Statistics (New York: John Wiley and Sons, Inc., 1971) pp. 206-215.

[2] Conover, pp. 216-223.

[3] Conover, pp. 206-215.

A record of the number of activities worked for each unit objective by each student was collected. This data was to be used as a measure of efficiency of learning.

Summary

The subjects were classified as ready or not ready on the basis of a readiness test designed by the investigator. The subjects were paired and one member of each pair was randomly assigned to a delayed symbolization group and the other member was assigned to an immediate symbolization group. The immediate symbolization groups of both ready and not ready subjects experienced a treatment in which written symbolization of addition and subtraction was introduced simultaneously with the concepts. The delayed symbolization group of ready subjects experienced a treatment in which written symbolization was delayed for five weeks. The delayed symbolization group of not ready subjects experienced a treatment in which written symbolization was delayed until the subjects were judged to be ready.

A posttest designed to measure subjects' meaningful learning was administered to all subjects. It was hypothesized that the ready subjects in the immediate symbolization group would demonstrate that the symbols were more meaningful to them than the subjects in the delayed symbolization group. It was also hypothesized that the not ready subjects in the delayed symbolization group would demonstrate that the symbols were more meaningful to them than the subjects in the immediate symbolization group.

Additional information was collected during the study for exploratory purposes. This information included audio and video tapes of many of the instructional sessions, subjects' records and instructors'

notes for each lesson. The use of this information in the exploratory aspects of the study will be discussed in Chapter V. The statistical analyses of the data, the results of the hypotheses testing, and the confidence intervals for each hypothesis will be presented in Chapter IV.

Chapter IV

ANALYSIS AND INTERPRETATION OF THE DATA

The purpose of this chapter is to describe the statistical analyses of the data and to describe the use of these analyses to test twelve null hypotheses related to meaningful learning of both ready and not ready subjects. A discussion of data collected concerning efficiency of learning and the analyses of data collected concerning missing addend problems will also be presented.

THE READY SUBJECTS

The Research Hypotheses

The research hypothesis related to the ready subjects is that if a student is ready for symbolization of addition and subtraction, he will learn the symbolization in a more meaningful way if the symbolization is introduced immediately rather than if it is delayed. The reasoning behind this hypothesis is related to the previously defined readiness state. If the attainment of the readiness state does effect the course of learning and the success of instruction, the learning of a student who has attained this readiness state would be more meaningful if the symbolization is introduced to him immediately. A delay in introduction would be inefficient.

In the present study, a subject's meaningful learning of the symbolization of addition and subtraction was assessed in terms of the

subject's ability to interpret, produce, and state answers to number sentences. The posttest used in the present study was designed as a measure of a subject's ability to interpret, produce, and state answers to number sentences; thus the posttest was a measure of a subject's meaningful learning of the written symbolization of addition and subtraction. It was assumed that Subject A demonstrated that the symbols of addition and subtraction were more meaningful to him than to Subject B if: (1) the total posttest score of Subject A was higher than the score of Subject B, or (2) the score of Subject A on one section of the posttest was higher than the score of Subject B, and the reverse was not true on any other section of the posttest. If the score of Subject A was higher than the score of Subject B on one section and the reverse was true on another section, only the total posttest score could be used to determine which subject demonstrated that the symbols were more meaningful to him.

Information concerning the research hypothesis was gained by testing the following null hypotheses using the data from the two groups of ready subjects:

(1) For students classified as ready on the basis of the readiness test, the time of introduction of symbolization will have no effect on students' ability to:

a) interpret addition number sentences.

b) interpret subtraction number sentences.

c) interpret addition and subtraction number sentences.

d) produce addition number sentences.

e) produce subtraction number sentences.

f) produce addition and subtraction number sentences.

g) state answers to addition number sentences.

h) state answers to subtraction number sentences.

i) state answers to addition and subtraction number sentences.

j) interpret, produce, and state answers to addition number sentences.

k) interpret, produce, and state answers to subtraction number sentences.

l) interpret, produce, and state answers to addition and subtraction number sentences.

<u>The Hypotheses Testing</u>

Scores on 12 sections of the posttest were recorded for each subject. Each of the 12 scores of a subject was related to one of the 12 null hypotheses. For example, the score related to Hypothesis 1-a was the number of addit[illegible] sentences correctly interpreted. The scores of each subject related to each hypothesis is included in Appendix F. For each hypothesis, the data consisted of eight pairs of scores, (X_1, Y_1). (X_2, Y_2), . . . , (X_8, Y_8), where X_i is the score of the i^{th} subject in the delayed symbolization group (RD), and Y_i is the score of the i^{th} subject in the immediate symbolization group (RI).

Each null hypothesis was tested by the Wilcoxon Signed Ranks Test procedure described by Conover.[1] In the model for this procedure, the population median of the differences, $D_i = Y_i - X_i$, is denoted $d_{.50}$. Each D_i has a probability of .5 of exceeding $d_{.50}$ and a probability of .5 of being less than $d_{.50}$. If $d_{.50} = 0$, then each D_i has a probability

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[1] W. J. Conover, <u>Practical Nonparametric Statistics</u> (New York: John Wiley and Sons, Inc., 1971) pp. 206-215.

of .5 of being positive, i.e., the Y_i score is larger, and a probability of .5 of being negative, i.e., the X_i score is larger.

The following are listed by Conover[2] as the assumptions of the model for the Wilcoxon Signed Ranks Test:

(1) Each D_i is a continuous random variable.

In practice, no measured random variable is continuous because of the finite capacity of measuring instruments. However, each D_i is a measure of a difference between two subjects' ability to interpret, produce, or state answers to number sentences. Ability is a continuous variable, thus each D_i is a measure of a continuous variable.

(2) The distribution of each D_i is symmetric.

Each D_i is a sample from a population distribution. Since one member of each pair of subjects in this study was randomly assigned to a treatment, each D_i has a probability of .5 of being less than zero and a probability of .5 of being greater than zero. Thus the probability distribution of each D_i is symmetric about zero.

(3) The D_i's are mutually independent.

The scores X_i and Y_i are mutually independent measures; thus each $D_i = Y_i - X_i$ is mutually independent.

(4) The D_i's all have the same median.

The probability distribution of each D_i has the median zero. Thus all the D_i's have the same probability distribution median, i.e., zero.

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[2] Conover, p. 207.

(5) The measurement scale of the D_i's is at least interval.

The distance between two D_i's may be expressed as a number of units; thus the measurement scale of the D_i's is at least interval.

The following statistical null and alternative hypotheses were used to test each of the twelve null hypotheses:

$$H_0: d_{.50} = 0$$

$$H_1: d_{.50} \neq 0$$

The alternative hypothesis may be stated as, "The values of the X_i's, the scores of the RD group, tend to be larger or smaller than the Y_i's, the scores of the RI group." If it were found that $d_{.50} < 0$, it would be assumed that values of the X_i's tended to be larger than the values the Y_i's. If it were found that $d_{.50} > 0$, it would be assumed that the values of the X_i's tended to be smaller than the values of the Y_i's.

Using the data for each of the hypotheses, the differences, $D_i = Y_i - X_i$, were computed for each pair of scores, and the D_i's were ranked without regard to sign. The test statistic T was the sum of the ranks of the positive D_i's and N was the number of D_i's not equal to zero. The test statistic T was compared with quantiles $W_{.025}$ and $W_{.975}$ of the Wilcoxon Signed Ranks Test statistic listed by Conover.[3] The value of N and the comparison quantiles $W_{.025}$ and $W_{.975}$ are presented in Table 7.

Summary

None of the twelve null hypotheses concerning the effect of the time of introduction of symbolization on ready students' ability to interpret, produce or state answers to addition or subtraction number

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[3] Conover, p. 393.

Table 7

The Ready Subjects

The Values of N, the Test Statistic T, and the Quantiles $W_{.025}$ and $W_{.975}$ for Each Null Hypothesis

Hypothesis	Task	N	T	$W_{.025}$	$W_{.975}$
1-a	Interpret +	3	3	0	6
1-b	Interpret −	4	8.5	0	10
1-c	Interpret +, −	6	16	1	20
1-d	Produce +	4	7.5	0	10
1-e	Produce −	4	3	0	10
1-f	Produce +, −	5	10.5	0	15
1-g	Answer +	6	13	1	20
1-h	Answer −	6	10	1	20
1-i	Answer +, −	8	20	4	32
1-j	Interpret, Produce, & Answer +	7	19	3	25
1-k	Interpret, Produce, & Answer −	8	22	4	32
1-l	Interpret, Produce, & Answer +, −	7	22	3	25

Decision Rule: Reject H_0: $d_{.50} = 0$ at level of significance $\alpha = .05$ if the value of T exceeds the value of $W_{.975}$ or is less than $W_{.025}$

sentences, or both, were rejected at the level of significance $\alpha = .05$. Also, none of the twelve hypotheses would have been rejected at the level of significance $\alpha = .10$.

As previously noted, a measure of a subject's ability to interpret, produce, and state answers to addition and subtraction number sentences was used to assess the subject's meaningful learning of the written symbolization of addition and subtraction number sentences. Therefore, the results of the hypothesis testing do not support the research hypothesis that the time of introduction of symbolization will affect ready students' meaningful learning of the written symbolization of addition and subtraction.

THE NOT READY SUBJECTS

The Research Hypotheses

The research hypothesis related to the not ready subjects is that if a student is not ready for symbolization of addition and subtraction, he will learn the symbolization in a more meaningful way if the symbolization is delayed until the student is ready. The reasoning behind this hypothesis is also related to the previously defined readiness state. If non-attainment of this readiness state does effect the course of learning and the success of instruction, the learning of a student who has not attained this readiness state should be more meaningful if the symbolization is delayed until the student is ready.

The same assumptions, those related to the assessment of the meaningful learning of Subject A versus Subject B, that were made for the ready subjects also applied to the not ready subjects. Information

about the research hypotheses was gained by testing the following null hypotheses:

(2) For students classified as not ready on the basis of the readiness test, the time of introduction of symbolization will have no effect on students' ability to:

(a) interpret addition number sentences.

(b) interpret subtraction number sentences.

(c) interpret addition and subtraction number sentences.

(d) produce addition number sentences.

(e) produce addition number sentences.

(f) produce addition and subtraction number sentences.

(g) state answers to addition number sentences.

(h) state answers to subtraction number sentences.

(i) state answers to addition and subtraction number sentences.

(j) interpret, produce, and state answers to addition number sentences.

(k) interpret, produce, and state answers to subtraction number sentences.

(l) interpret, produce, and state answers to addition and subtraction number sentences.

The Hypotheses Testing

Scores on 12 sections of the posttest were recorded for each subject. As with the ready subjects, each score was related to one of the 12 null hypotheses. Thus for each hypothesis, the data consisted of eleven pairs of scores, (X_1, Y_1), (X_2, Y_2), . . . , (X_{11}, Y_{11}), where X_i is the score of the i^{th} subject in the delayed symbolization group

(ND), and Y_i is the score of the i^{th} subject in the immediate symbolization group (NI).

Again, each null Hypothesis was tested by the Wilcoxon Signed Ranks Test procedure described by Conover.[4] The following statistical null and alternative hypotheses were used to test each of twelve null hypotheses:

$$H_0: d_{.50} = 0$$

$$H_1: d_{.50} \neq 0$$

The same procedure used to obtain the test statistic T and the value of N for the ready subjects was used for the not ready subjects. The value of the test statistic T, the value of N, and the comparison quantiles, $W_{.025}$ and $W'_{.975}$ are presented in Table 8.

Summary

The null hypotheses 2-a, 2-b, and 2-c were rejected at the level of significance $\alpha = .05$. The small values of the test statistic T for these hypotheses, i.e., the value of T was less than $W_{.025}$, indicated that $d_{.50} < 0$. This implies that the values of the X_i's, the scores of the ND group, tended to be larger than the values of the Yi's, the scores of the NI group. These results imply that with students classified as not ready, the time of introduction of symbolization had an effect on students' ability to:

(a) interpret addition number sentences.

(b) interpret subtraction number sentences.

(c) interpret addition and subtraction number sentences.

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[4] Conover, pp. 206-215.

Table 8

The Not Ready Subjects

The Values of N, the Test Statistic T, and the Quantiles $W_{.025}$ and $W_{.975}$ for Each Null Hypothesis

Hypotheses	Task	N	T	$W_{.025}$	$W_{.975}$
2-a	Interpret +	8	3.5*	4	32
2-b	Interpret −	10	6*	9	46
2-c	Interpret +, −	11	7*	11	53
2-d	Produce +	9	14.5	6	39
2-e	Produce −	9	8**	6	39
2-f	Produce +, −	10	11.5	9	46
2-g	Answer +	8	20	4	32
2-h	Answer −	8	20	4	32
2-i	Answer +, −	7	16.5	3	25
2-j	Interpret, Produce, & Answer +	10	17.5	9	46
2-k	Interpret, Produce, & Answer −	10	9.5**	9	46
2-l	Interpret, Produce, & Answer + −	11	13**	11	53

Decision Rule: Reject $H_0: d_{.50} = 0$ at level of significance $\alpha = .05$ if the value of T exceeds the value of $W_{.975}$ or is less than the value of $W_{.025}$.

*$H_0: d_{.50} = 0$ rejected at level of significance $\alpha = .05$

**$H_0: d_{.50} = 0$ would have been rejected at level of significance $\alpha = .10$

ERIC

The null hypotheses 2-d, 2-e, 2-f, 2-g, 2-h, 2-i, 2-j, 2-k, and 2-l were not rejected at the level of significance $\alpha = .05$. Hypotheses 2-e, 2-k, and 2-l were rejected at level of significance $\alpha = .10$.

As previously noted, a measure of a subject's ability to interpret, produce, or state answers to addition and subtraction number sentences was used to assess the subject's meaningful learning of the written symbolization of addition and subtraction. Although the time of introduction of symbolization did not have a statistically significant effect on not ready students' ability to produce or state answers to number sentences, it did have a statistically significant effect on students' ability to interpret number sentences. Thus, the results of the hypothesis testing support, in part, the research hypothesis that the time of introduction of symbolization will affect not ready students' meaningful learning of the written symbolization of addition and subtraction.

CONFIDENCE INTERVALS

Confidence intervals were also constructed for each of the twelve null hypotheses of both ready and not ready subjects. The confidence intervals were constructed by a geometric construction procedure described by Conover.[5] Intervals formed in this way have a probability of $1-\alpha$ of containing the true parameter $d_{.50}$, the population median of the D_i's.

As previously described, the data for each hypothesis consisted of pairs of scores, (X_i, Y_i), where X_i is the score of the i^{th} subject

[5] Conover, pp. 216-223.

in the delayed symbolization and Y_i is the score of the i^{th} subject in the immediate symbolization group. The differences, $D_i = Y_i - X_i$, were computed for each pair of scores, and the population median of the differences was denoted $d_{.50}$. The model for the confidence intervals asserts that if $d_{.50} = 0$, each D_i has a probability of .5 of being positive, i.e., the Y_i score is larger, and a probability of .5 of being negative, i.e., the X_i score is larger. If $d_{.50} < 0$, it would be assumed that the values of the X_i's tended to be larger than the values of the Y_i's. If $d_{.50} > 0$, it would be assumed that the values of the X_i's tended to be smaller than the values of the Y_i's. The assumptions of the model are the ~~same as~~ the assumptions for the Wilcoxon Signed Ranks Test.

Confidence intervals were constructed that have a probability of 95% and a probability of 90% of containing the true parameter $d_{.50}$, i.e., $p\ (L \leq d_{.50} \leq U) = .95$ and $p\ (L \leq d_{.50} \leq U) = .90$. The upper and lower bounds, L and U, of these confidence intervals for each of the twelve hypotheses relating to ready students are listed in Table 9.

All of these confidence intervals, both the 90% and 95% intervals, contained zero. This was consistent with the fact that none of the twelve null hypotheses was rejected. However, the lower bounds of three confidence intervals were zero. These confidence intervals were the 90% interval related to interpretation of subtraction number sentences, Hypothesis 1-b, and both the 90% and 95% confidence intervals related to production of addition number sentences, Hypothesis 1-d. However, confidence intervals were constructed for these hypotheses that have a probability of .80 of containing parameter $d_{.50}$, i.e., $P\ (L \leq d_{.50} \leq U) = .80$, and the lower bounds were still found to be zero.

Table 9

The Ready Subjects

Upper and Lower bounds of 95% and 90% Confidence Intervals for the Parameter d'_{50} of the Data for Each Hypothesis

Hypothesis	Task	95%		90%	
		Upper Bound	Lower Bound	Upper Bound	Upper Lower
1-a	Interpretation +	.50	-1.00	.50	-1.00
1-b	Interpretation -	2.00	- .50	1.50	0
1-c	Interpretation +, -	2.00	- .50	2.00	-1.50
1-d	Production +	1.00	0	1.00	0
1-e	Production -	.50	- .50	.50	-.50
1-f	Production +, -	1.50	-1.00	1.00	- .50
1-g	Answers +	1.00	-1.00	1.00	- .50
1-h	Answers -	3.00	-2.00	2.50	-1.50
1-i	Answers +, -	3.00	-1.50	2.00	-1.50
1-j	Interpretation, Production, & Answers +	2.50	-2.00	2.00	-2.00
1-k	Interpretation, Production, & Answers -	5.50	-2.50	4.50	-1.50
1-l	Interpretation, Production, & Answers +, -	6.00	-3.00	5.00	-1.00

The upper and lower bounds of the 95% and 90% confidence intervals for each of the twelve hypotheses relating to not ready students are listed in Table 10. The 90% and 95% intervals relating to the interpretation of addition and subtraction number sentences, Hypotheses 2-a, 2-b, and 2-c do not contain zero. This is consistent with the fact that these hypotheses were rejected at the level of significance $\alpha = .05$. These confidence intervals also indicated that the probability is .95 that $d_{.50} < 0$. A value of $d_{.50} < 0$ implies that the values of the X_i's, the scores of the subjects in the ND group, tended to be larger than the values of the Y_i's, the scores of the subjects in the NI group.

The 90% confidence intervals relating to hypotheses 2-e, 2-k, and 2-l have an upper limit of 0. Confidence intervals were constructed for these hypotheses that have a probability of .80 of containing the parameter $d_{.50}$, i.e., $P(L \leq d_{.50} \leq U) = .80$. The upper and lower limits of these intervals are listed in Table 11.

Table 11

80% Confidence Intervals for Hypotheses 2-e, 2-k, and 2-l

Hypothesis	U	L
2-e	-.50	-2.50
2-k	-1.00	-4.00
2-l	-1.00	-6.50

The 80% confidence intervals do not contain zero. This is consistent with the fact that these hypotheses would have been rejected at the

Table 10

The Not Ready Subjects

Upper and Lower Bounds of 95% and 90% Confidence Intervals for the Parameter $d_{.50}$ of the Data for Each Hypothesis

Hypothesis	Task	95% Upper Bound	95% Lower Bound	90% Upper Bound	90% Lower Bound
2-a	Interpretation +	-.50	-1.50	-.50	-1.50
2-b	Interpretation -	-.50	-2.00	-.50	-2.00
2-c	Interpretation +, -	-.50	-3.50	-.50	-3.00
2-d	Production +	1.00	-2.50	.50	-2.50
2-e	Production -	0	-3.00	0	-3.00
2-f	Production +, -	.50	-4.00	0	-4.00
2-g	Answers +	1.00	-.50	1.00	-.50
2-h	Answers -	.75	-1.00	.75	-1.00
2-i	Answers +, -	2.00	-1.00	1.50	-1.00
2-j	Interpretation, Production, & Answers +	1.00	-4.00	1.00	-3.50
2-k	Interpretation, Production, & Answere -	0	-5.00	0	-4.50
2-l	Interpretation, Production, & Answers +, -	.50	-8.00	0	-7.50

level of significance $\alpha = .10$. These confidence intervals also indicated that the probability is .80 that $d_{.50} < 0$. A value of $d_{.50} < 0$ implies that values of the X_i's, the scores of the ND group, tended to be larger than the values of the Y_i's, the scores of the NI group.

Except for hypotheses 2-g and 2-1, the confidence intervals for each hypothesis are centered to the right of zero, i.e., the negative direction. This is indicative of the consistently larger values of the X_i's, the scores of the ND group.

MISSING ADDEND PROBLEMS

The data for the six missing addend number sentences on the posttest were analyzed separately from the data for the other items on the posttest. The same method, the Wilcoxon Matched Pairs Signed Ranks procedure described by Conover,[6] that was previously used in hypothesis testing was used to test the null hypotheses:

3-a: The time of introduction of symbolization will have no effect on ready subjects' ability to state answers to and produce missing addend number sentences.

3-b: The time of introduction of symbolization will have no effect on not ready subjects' ability to state answers to and produce missing addend number sentences.

Again, the following statistical null and alternative hypotheses were used to test these research hypotheses at the level of significance $\alpha = .50$:

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[6] Conover, pp. 206-215.

$$H_0: d_{.50} = 0$$

$$H_a: d_{.50} \neq 0$$

The values of N, the test statistic T, and the comparison quantiles $W_{.025}$ and $W_{.975}$ are listed for each hypothesis in Table 12.

Table 12

Missing Addend Number Sentences

The Values of N, the Test Statistic T, and the Quantiles $W_{.025}$ and $W_{.975}$ for Each Hypothesis

Hypothesis	Group	N	T	$W_{.025}$	$W_{.975}$
3-a	Ready	5	5	0	15
3-b	Not Ready	10	9**	9	46

**Would have been rejected at the level of significance $\alpha = .10$.

The null hypotheses were not rejected at the level of significance $\alpha = .05$. Hypothesis 3-b, relating to the not ready subjects, was rejected at the level of significance $\alpha = .10$.

A MEASURE OF EFFICIENCY OF LEARNING

As noted in Chapter III, the unit developed for the present study consisted of 63 activities for addition and subtraction. The activities were not strictly sequenced and the instructions were to use only as many activities as needed for student mastery of each objective of the unit. The subjects' worksheets and instructors' records of which activities were covered by each subject were collected. It was planned that the number of activities needed by each subject for mastery of each

objective would be used as measure of efficiency of learning. The data indicated that the only differences in the number of activities completed by the subjects were among instructors. However, audio and video tapes of some of the instructional sessions of the study, teachers' records, and students' worksheets were also collected. This data was used to draw some conclusions about the effect of time of introduction of symbolization on efficiency of learning. The conclusions will be discussed in Chapter V.

SUMMARY

Twelve null hypotheses related to meaningful learning were tested for both ready and not ready subjects. None of the twelve null hypotheses concerning the effect of the time of introduction of symbolization on ready students' ability to interpret, produce, or state answers to addition or subtraction number sentences, or both, were rejected. The three null hypotheses concerning the effect of time of introduction of symbolization on not ready students' ability to interpret number sentences were rejected at the level of significance $\alpha = .05$. The three null hypotheses concerning the effect of time of introduction of symbolization on not ready students' production of number sentences were rejected at the level of significance $\alpha = .10$.

The conclusions based on the results of the study, the limitations of the study, and suggestions for additional research are discussed in Chapter V.

Chapter V

SUMMARY, CONCLUSIONS, LIMITATIONS, AND SUGGESTIONS FOR FURTHER RESEARCH

This chapter presents a summary of the study and conclusions based on the results of the study. The limitations of the study and suggestions for further research are also presented in this chapter.

SUMMARY OF THE STUDY

The purpose of this study was to investigate the relationship among children's understanding of mathematical concepts, written symbolization of these concepts, and a well defined "readiness" for written symbolization based on verbal facility with the concepts to be symbolized. It was hypothesized that readiness for written symbolization, defined in terms of verbal facility, would influence the course of learning and the success of instruction. This general hypothesis was tested through a teaching experiment on addition and subtraction concepts and the symbols that express them for small whole numbers at the first grade level.

The subjects were 38 first grade students at David C. Barrow Elementary School, Athens, Georgia. The teaching experiment began in September, 1975. At that time the first grade students had worked only with numeration and had not been introduced to addition and subtraction concepts.

In this study, readiness for written symbolization of a topic was defined as follows:

> Given a topic in elementary mathematics, there are sets of objectives, the attainment of which indicates mastery of the topic. Omitting those objectives concerned with reading, writing, or speed of response, a child is ready for the introduction of written symbolization of the topic when he has mastered the objectives of the topic verbally, perhaps with the aid of pictures or manipulatives.

The definition of readiness was then applied to the topic of the study based on the set of objectives for addition and subtraction used at Barrow School.

The subjects were classified as ready or not ready according to scores on a readiness test based on the definition of readiness applied to addition and subtraction. The subjects were then paired by means of the readiness test scores and Key Math Test scores. This pairing procedure resulted in eleven pairs of not ready subjects and eight pairs of ready subjects. One member of each pair was randomly assigned to an immediate symbolization group and the other member was assigned to a delayed symbolization group.

All subjects received twelve weeks of instruction on introductory addition and subtraction. The instructional unit consisted of 63 activities for nine objectives on addition and subtraction. Each activity was divided into the following three sections:

(1) Concrete pictorial (CP): Lessons in the concrete-pictorial sections involved the use of concrete objects or pictures. The lessons were designed to be conducted in a verbal mode with

emphasis on verbalization on the part of the teacher and the students as a group. Written symbols were not included in these sections.

(2) Verbal (V): Lessons in the verbal sections involved oral responses primarily on the part of the students. The lessons were designed to encourage the students to say number sentences when given actions or pictures illustrating the number sentences, and to interpret given number sentences by describing or demonstrating actions appropriate for the number sentences.

(3) Symbolic (S): Lessons in the symbolic sections involved written symbols. These lessons were designed to be similar to pages typical of first grade arithmetic texts.

The immediate symbolization group of both ready and not ready subjects experienced a treatment in which written symbolization was introduced simultaneously with the concepts. The subjects in these treatments worked all three sections, CP, V, and S, of each activity. These groups were essentially control groups since it is the usual practice in the first grade to introduce written symbolization of a concept at the time of the introduction of the concept.

The delayed symbolization group of ready subjects experienced a treatment in which written symbolization was delayed for five weeks. The subjects in this treatment worked only the CP and V sections of each activity during the five week delay. At the end of the delay period, the subjects worked the previously skipped S sections of each activity and then worked all three sections of each additional activity.

The not ready subjects in the delayed symbolization group experienced a treatment in which written symbolization of each of the nine

objectives of the unit was delayed for each subject until the subject had mastered each of the objectives verbally. During the delay period, these subjects worked only the CP and V sections of each activity. When a subject had mastered an objective verbally, he then worked the previously skipped S sections of the activities for that objective. The subject then proceeded to the activities for the next objective, but worked only the CP and V sections until the objective was mastered verbally.

The subjects were given a posttest at the conclusion of the treatments. The posttest was designed to measure the subjects' ability to interpret given number sentences by describing or demonstrating actions appropriate for the number sentences, to produce number sentences when given actions on sets, and to state answers to given number sentences. Since, in the present study, a student's meaningful learning of the symbolization of addition and subtraction was defined in terms of his ability to interpret, produce, and state answers to number sentences, the posttest was a measure of a subject's meaningful learning of written symbolization of addition and subtraction.

If a student is not ready for the introduction of symbolization, a delay of symbolization until the student is ready should facilate learning. Therefore, it was hypothesized that with students classified as not ready, those who experience a delay of symbolization until they are determined to be ready will be better able to interpret, produce, and state answers to number sentences than students who experience immediate introduction of symbolization.

If a student is ready for the introduction of symbolization, immediate introduction of symbolization should facilate learning.

Therefore, it was hypothesized that with students classified as ready, those who experience immediate introduction of symbolization will be better able to interpret, produce, and state answers to number sentences than students who experience a delay of symbolization.

These two hypotheses were tested using the Wilcoxon Signed Ranks Test procedure described by Conover[1]. At the level of significance $\alpha = .05$, there were no significant differences between the scores on any section of the posttest of the two groups of ready subjects. There was a significant difference between the scores on the interpretation section of the posttest of the two groups of not ready subjects. The subjects in the delayed symbolization group had significantly higher ($\alpha < .05$) scores on the interpretation section of the posttest than subjects in the immediate symbolization group. There were no significant differences between the scores on the production and answer section of the posttest between the two groups of not ready subjects.

The results of the data analyses do not support the hypothesis that the time of introduction of symbolization will affect ready students' meaningful learning of written symbolization of addition and subtraction. The results do support, in part, the hypothesis that the time of introduction of symbolization will affect not ready students' meaningful learning of written symbolization of addition and subtraction.

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[1] W. J. Conover, Practical Nonparametric Statistics (New York: John Wiley and Sons, Inc., 1971), pp. 206-215.

OBSERVATIONS

The Ready Subjects

The investigator's review of audio and video tapes of sessions of the delayed symbolization group indicated that the subjects became bored and frustrated during the delay period. Many of these subjects were writing number sentences on their own. At the end of the delay period, the end of the fifth week of the treatments, the subjects in the delayed symbolization group had covered activities for two more of the nine unit objectives than subjects in the immediate symbolization group. However, the subjects in the delayed symbolization group were skipping the symbolic sections of each activity. During the sixth and seventh weeks of the treatments, these subjects worked the previously skipped symbolic sections, and at the beginning of the eighth week of the treatments, these groups were essentially working activities for the same objectives. The investigator's review of audio and video tapes of the eighth through the twelfth weeks of the treatments did not detect any differences between the performance of the two groups of ready subjects. Thus the investigator's review of tapes and the results of the data analyses supported the following conclusion:

> Conclusion 1: The time of introduction of symbolization does not affect ready students' meaningful learning of the symbolization of addition and subtraction.

The following conclusion is supported only by the investigator's review of tapes:

> Conclusion 2: A delay of symbolization may cause symptoms of boredom and frustration among ready students.

The Not Ready Subjects

Although the time of introduction of symbolization did not have a statistically significant effect on not ready subjects' ability to produce or state answers to number sentences, it did have a statistically significant, $\alpha = .05$, effect on not ready subjects' ability to interpret number sentences. Although the results were not significant on the production section of the posttest, the scores of the subjects in the delayed symbolization group were consistently higher than the scores of the subjects in the immediate symbolization group. The scores of the subjects in the two groups were about the same on the answer section of the posttest.

The investigator's review of audio and video tapes of the lessons of the not ready groups and examination of students' worksheets indicated that the delay of symbolization facilated learning. When the subjects in the delayed symbolization group had mastered an objective verbally, and were given a symbolic section worksheet, they seemed to require less instruction and help from the teacher than the subjects in the immediate symbolization group.

The following conclusion is supported by the investigator's review of tapes and is supported in part by the data analyses:

Conclusion 3: The time of introduction of symbolization does affect not ready students' meaningful learning of the symbolization of addition and subtraction.

The following conclusion is supported only by the investigator's review of tapes:

Conclusion 4: The learning of written symbolization by not ready students is more efficient if the symbolization is delayed until the students are ready.

Since the results of the data analyses favored the subjects in the delayed symbolization group and the investigator's review of tapes indicated that the delay of symbolization facilitated learning of written symbolization, the investigator concluded the following:

Conclusion 5: If a student is not ready for the introduction of symbolization of addition and subtraction, the student's learning of the symbolization will be more meaningful if the symbolization is delayed until the student is ready.

Readiness - Meaningful Learning

The purpose of this study was to investigate whether children's readiness, as defined in this study, influences the efficiency of learning and the meaningfulness of learning the written symbolization of addition and subtraction by first grade children. The influence was hypothesized to act in the following way:

(1) Children who are not ready should learn more efficiently and the learning be more meaningful if the symbolization is delayed until they are ready:

(2) Children who are ready should learn more efficiently and the learning be more meaningful if the symbolization is introduced immediately.

The hypothesized influence of children's readiness on children's meaningful learning was supported for children who were not ready. The delay of symbolization did not affect the meaningful learning of ready

children. However, in the opinion of the investigator, the evidence of the influence of children's readiness on not ready children's meaningful learning is sufficient to conclude the following:

Conclusion 6: Children's readiness, as defined in this study, influences the meaningfulness of learning the written symbolization of addition and subtraction.

Readiness - Efficiency of Learning

The number of activities completed by each student for each unit objective was to be used as a measure of efficiency of learning. The only differences found in the number of activities completed were among instructors. However, the investigator's review of audio and video tapes of lessons indicated that readiness, as defined in this study, did influence the efficiency of learning; the delayed group of ready subjects experienced boredom and frustration (Conclusion 2), and the learning of symbolization by the not ready subjects in the delayed group was more efficient (Conclusion 4). Therefore, the following conclusion is supported only by the investigator's review of tapes:

Conclusion 7: Children's readiness, as defined in this study, influences the efficiency of learning the written symbolization of addition and subtraction.

The Missing Addend Problems

The results of the data analyses did not support the hypothesis that time of introduction of symbolization will affect ready or not ready students' ability to produce or state answers to missing addend number sentences. However, although the results were not significant at the level of significance $\alpha = .05$, the scores on the missing addend number

sentences of the not ready subjects in the delayed symbolization group were significantly higher at the level of significance $\alpha = .10$ than the scores of the not ready subjects in the immediate symbolization group. The subjects in the present study were not introduced to missing addend number sentences in any of the four treatments. Therefore, any conclusions formed on the basis of the data for the missing addend number sentences would involve transfer of learning. The investigation of the relationship among readiness for learning written symbolization, the time of introduction of symbolization, and the transfer of the learning will be left for future research.

Implications of the Study

Pending the limitations of the study and supposing further confirmation of the results of the study by parallel or replication studies, the study may have implications for mathematics education.

The mathematics curriculum of kindergarten and primary programs would have increased emphasis on the development of concepts and the associated vocabulary that will be needed in the primary grades. Concrete models would be used to stimulate discussion and emphasis would be placed on building the child's spoken vocabulary.

In the primary grades, a topic would be introduced and the concepts and vocabulary extensively developed before written symbolization would be introduced. This would suggest a reorganization of traditional text material with increased emphasis on verbal activities suggested in the teacher's edition. Totally individualized programs, such as the Individually Prescribed Mathematics (IPI) program, would need reorganization to build in the verbal development needed prior to symbolization.

Many teachers presently group students in first grade; customary groups are a "readiness" group doing essentially kindergarten work and one or more groups doing first grade work. The results of this study suggest that verbal facility might be a criterion for placing students in groups. This criterion could be used topic by topic for moving a child from readiness work to more formal mathematics.

Teachers should provide for more classroom interaction and discussion, especially activities in which the children use verbally the mathematical words they will later symbolize. This interaction and discussion appears to be desirable to develop the necessary vocabulary in mathematics. Ideally, the balance between written and verbal activities in the classroom should change with increased emphasis on verbal activities prior to written activities. Skills of planning and conducting verbal mathematical activities should be specifically identified and developed in teacher education programs.

LIMITATIONS OF THE STUDY

The purpose of this study was to investigate the relationships among children's understanding of mathematical concepts, written symbolization of these concepts, and a specifically defined readiness for written symbolization based on verbal facility with the concepts to be symbolized. However, the study dealt specifically with the mathematical concepts of addition and subtraction of small whole numbers at the first grade level. This limits the generalizability of the conclusions of the study to other topics in mathematics and to other grade levels.

A second limitation of the study was that all subjects were drawn from the same school and that instructors were not randomly assigned to

treatment groups. The assignment of teachers to treatment groups was done for the convenience of the four regular first grade teachers at Barrow School. This limits generalizability of the results.

A third limitation of the study was the small sample sizes. Although the readiness test was administered to all first grade students at Barrow School, only 17 subjects were found to be ready. For this reason, only 8 pairs of ready subjects comprised that sample. There were 53 subjects classified as not ready on the basis of the readiness test. However, the restrictions imposed on the pairing procedure by the investigator only allowed for a match of 11 pairs.

SUGGESTIONS FOR FURTHER RESEARCH

One of the limitations of the present study was that the study dealt directly with only one topic in elementary mathematics at one grade level, i.e., addition and subtraction of small whole numbers at the first grade level. However, in the present study, readiness for introduction of symbolization was first defined for any topic in elementary mathematics. This definition could be applied to other topics in elementary mathematics at other grade levels. The present study thus provides a basis for parallel studies concerned with readiness for the introduction of written symbolization.

It is suggested that replication studies be made with the following modifications:

(1) It is suggested that teachers be randomly assigned to treatment groups. A random assignment of teachers would increase the reliability of the results.

(2) Since the only significant differences found in the data analyses were between the groups of not ready subjects, it is recommended that future studies concerning the effect of readiness for symbolization on students' meaningful learning of the symbolization concentrate on not ready students.

(3) Even when significant differences between the scores of the two groups of not ready subjects were not found, the posttest scores of the subjects in the delayed symbolization group were consistently higher than the posttest scores of subjects in the immediate symbolization group. For this reason, it is suggested that one-tailed statistical tests be employed in future replication studies instead of the two-tailed statistical tests used in the present study.

It was the opinion of the investigator, before the beginning of the present study, that many students are introduced to written mathematical symbolization before they are ready. The study described in this dissertation was designed to investigate that opinion. Readiness for written symbolization was defined and the relationship between that readiness and the learning of written symbolization was investigated. It is the role of future studies to provide more information on the relationship between readiness for written mathematical symbolization and the meaningful learning of the written symbols.

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APPENDIX A

OBJECTIVES FOR READINESS TEST

READINESS TEST ITEMS

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OBJECTIVES FOR READINESS TEST

(1) Given objects or a picture that illustrate the union of two sets, the student states the sum and says the number sentence.

(2) Given a collection of objects, and given verbally an addition number sentence, a + b = c, the student forms sets having a and b elements respectively and illustrates the union of the sets and states the sum.

(3) Given objects or a picture that illustrate the removal or partitioning of a subset, the student states the difference and says the number sentence.

(4) Given a collection of objects and given verbally a subtraction number sentence, a - b = c, the student forms a set having a elements and removes or partitions a subset having b elements and states the difference.

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ITEMS FROM KEY MATH TEST

D-1 One match and two matches are how many matches?

picture: Objective 1

D-2 There are three birds. Two more join them. How many birds?

picture of five birds Objective 1

D-3 Two frogs are joined by four more frogs. How many frogs are there now?

picture of six frogs Objective 1

E-1 This card has three buttons. If you took one button away, how many would be left?

picture: Objective 2

E-2 This card has five green buttons. If you took two buttons away, how many would be left?

picture: Objective 2

E-3 This card has eight red buttons. If you took four buttons away, how many would be left?

picture: Objective 2

J-1 John had two cookies. His dog ate one cookie. How many cookies does John have left?

picture of boy with cookie and dog Objective 2

J-2 Brian had three marbles. He finds two more. How many marbles does he have now?

picture of bag with three marbles and hand with two marbles Objective 2

J-3 Tom went to the store once. Nancy went to the store twice. How many trips were made to the store?

picture of two identical girls going to a store and one boy going to a store Objective 1

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Items from P.M.D.C.

(13) "I have three pencils. You have two pencils. How many pencils do we have together?"

Objective 1

(27) "I had seven toy cars. You took three toy cars. How many toy cars do I have now?"

Objective 2

(31) Point to the tunnel. Say, "This is a tunnel. There are three cars outside the tunnel and there are six cars still in the tunnel. How many cars are there all together?"

Objective 1 and 2 Picture: Tunnel with three cars outside it.

(43) "Together we have six pennies. You have four pennies. How many pennies do I have?"

Objective 1 and 2

TEST PREPARED BY INVESTIGATOR

DO	SAY
I. Give the child picture card 1. (picture of three bugs and two bugs)	
a. Point to one circle.	a. How many bugs are in this circle?
b. Point to the other circle.	b. How many bugs are in this circle?
c. Point to the whole card with a sweeping motion.	c. How many bugs in all?
	d. Tell me a number sentence for this card.
Objective 1	e. Probe 1: Tell me a number sentence that uses <u>plus</u> and <u>equals</u>.
	f. Probe 2: Do you know what a number sentence is? I am saying a number sentence when I say, "One plus one equals two." or "Two plus two equals four." Can you tell me a number sentence for this picture?
II. Give the child picture card 2. (picture of four bugs)	
a. Point to the card.	a. How many bugs in all?
	b. How many bugs are leaving?
	c. How many bugs are left?
	d. Tell me a number sentence for this card.
Objective 2	e. Probe 1: Tell me a number sentence that uses <u>minus</u> and <u>equals</u>.
	f. Probe [illegible] Do you know what a number sentence is? I am saying a number sentence when I say, "Three minus one equals two." Can you tell me a number sentence for this picture?

DO	SAY
III. Give the child nine counters.	
a. Point to the counters.	a. Can you use the counters to show me how much is two plus two?
	b. Repeat for three plus two.
	c. Repeat for four minus one.
Objectives 3 & 4	d. Repeat for five minus three.
IV. Give the child picture card 3. (picture of five birds in a tree)	
a. Point to the card.	a. How many birds are in the tree?
	b. If two birds come, how many birds would there be?
Objective 1	
V. Give the child picture card 4. (picture of six birds in a tree)	
a. Point to the card.	a. How many birds are in the tree?
Objective 2	b. If two birds fly away, how many birds would be left?.

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APPENDIX B

OBJECTIVES FOR THE UNIT AND CORRESPONDING

IPI OBJECTIVE NUMBERS

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INTRODUCTORY ADDITION AND SUBTRACTION

The activities in this unit are based on the following objectives:

Activities	IPI Objective	Objective (Limit: maximum of 9 elements, sums through 9)
1 - 10 21 - 30	AS 4,5,12,13	1. Given two sets of objects and an illustration of the union of the sets, or a picture illustration the union of the two sets, the student can say (write) the number sentence.
11 - 20 21 - 30	AS 4,5,12,13	2. Given a set of objects and an illustration of the removal of a subset, or a picture illustrating the removal of a subset, the student can say (write) the number sentence.
31 - 40	AS 7	3. Given a spoken (written) addition number sentence, the student can demonstrate or describe the union of sets appropriate to the number sentence.
31 - 40	AS 7	4. Given a spoken (written) subtraction number sentence, the student can demonstrate or describe the removal of a subset appropriate to the number sentence.
41 - 45	AS 8,9	5. Given a number line illustrating an addition or subtraction number sentence, the student can say (write) the addition or subtraction number sentence.
41 - 45	AS 10,11	6. Given a spoken (written) addition or subtraction number sentence, the student uses the number line to illustrate the number sentence.
46 - 51	AS 14	7. Given a spoken (written) addition number sentence, the student says (writes) a subtraction sentence using the same numbers.
52 - 60	AS 15	8. Given a spoken (written) number sentence of the form, a + __ = c or a - __ = c, the student finds the answer.
61 - 63	AS 16,17	9. Given a number (spoken or written) the student says (writes) an addition sentence in which the number is the sum, or a subtraction sentence in which the number is the difference.

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APPENDIX C

SAMPLES FROM THE TEACHER'S MANUAL

SAMPLE WORKSHEETS.

CP_2

ACTIVITY 2

Materials: Toy barn, toy animals.

Place three animals in the barn. Ask: "How many animals are in the barn?" Place two animals in the barn, ask: "How many animals are in the barn now?" Tell the children the number sentence, three plus two equals five. Let the children say the number sentence together. Repeat for other numbers.

No Worksheet

V_2

Materials: Toy barn, toy animals, math-a-phone.

Give each child a math-a-phone. Explain that one can only say number sentences on a math-a-phone. Tell each child to call his parents or a friend. Repeat CP_2 ; let each child tell his parents or friend what happened using a number sentence.

(mastery indicative of readiness obj. 1)

No Worksheet

S_2

Materials: Worksheet, pencil.

Circle the correct number sentence:

Worksheet S-2

2 + 2 = 4   3 + 2 = 5   1 + 1 = 2

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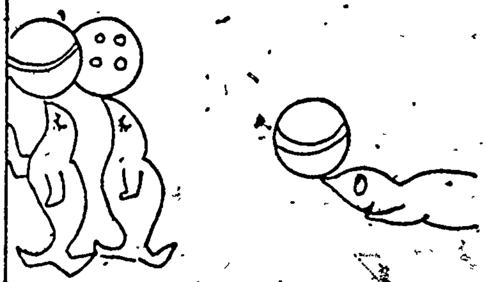
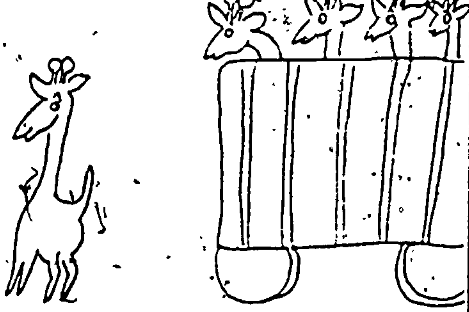
Obj. 1 (AS 4,5)
Sheet # S-2
Name ______________
DATE ______________

NAME ______________

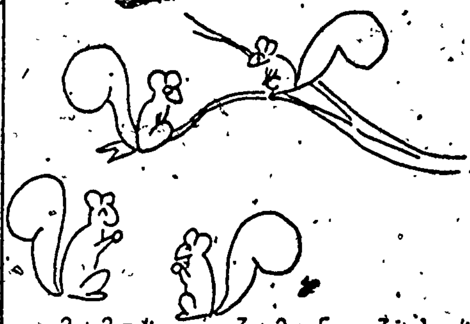
2 + 2 = 4   3 + 2 = 5   1 + 6 = 7



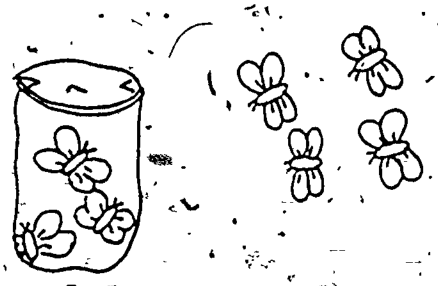
2 + 2 = 4   3 + 3 = 6   2 + 1 = 3



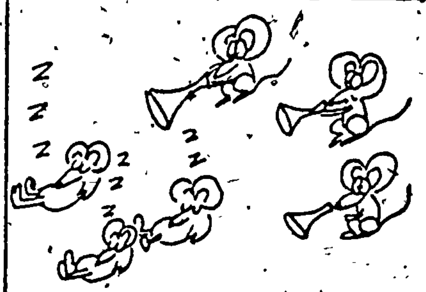
1 + 2 = 3   1 + 4 = 5   1 + 7 = 8



2 + 2 = 4   3 + 2 = 5   3 + 1 = 4



3 + 3 = 6   3 + 4 = 7   5 + 2 = 7



3 + 3 = 6   3 + 4 = 7   5 + 1 = 6

CP_{12}

ACTIVITY 12

Materials: Toy barn, toy animals.

Place five animals in the barn. Ask: " How many animals are in the barn? " Remove two animals from the barn and say: " Two animals leave, how many are left? " Say the number sentence for the children. Let the children say the number sentence together. Repeat for other numbers.

No Worksheet

V_{12}

Materials: Toy barn, toy animals.

Repeat CP_{12}. Vary by allowing children to manipulate animals. Let each child say a number sentence. Say the number sentences together.

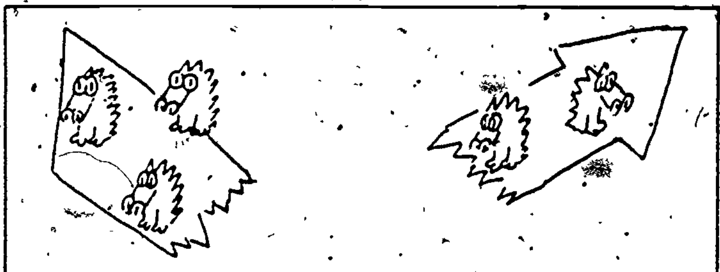
(mastery indicative of readiness obj 2)

No Worksheet

S_{12}

Materials: Worksheet, pencil.

Circle the correct number sentence:

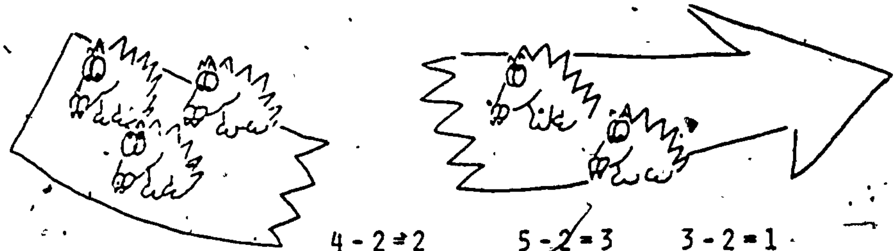


2 - 1 = 1   2 - 2 = 0   5 - 2 = 3

Worksheet S-12

Obj. 2 (AS 4,5)
Sheet # S-12
Name ______________
Date ______________

NAME ______________

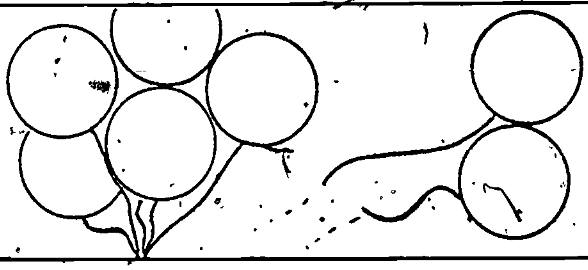


4 - 2 = 2 5 - 2 = 3 3 - 2 = 1

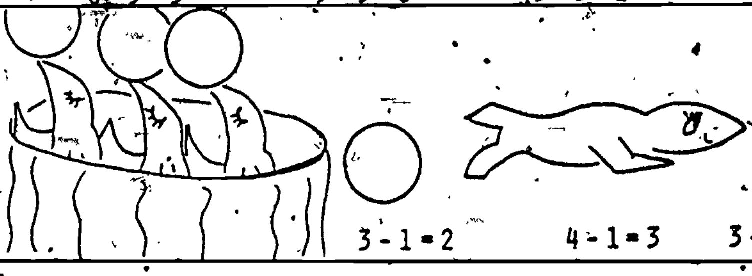
5 - 2 = 3

7 - 2 = 5

7 - 1 = 6



6 - 3 = 3 3 - 3 = 0 5 - 3 = 2



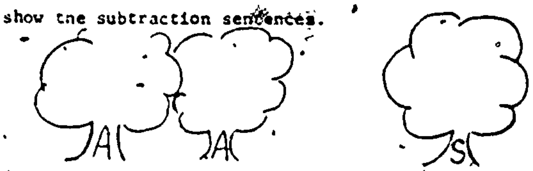
3 - 1 = 2 4 - 1 = 3 3 - 2 = 1

CP_{34}

ACTIVITY 34

Materials: Worksheet, pencil

Tell the children an addition or subtraction number sentence. Ask the children to draw apples on each addition tree to show the addition sentence and to draw and mark off apples on each subtraction tree to show the subtraction sentences.



(mastery indicative of readiness obj. 3 or obj. 4 or both)

Worksheet CP-34

V_{34}

Materials: none.

Tell each child an addition or subtraction number sentence. Ask each child to tell a story using the number sentence.

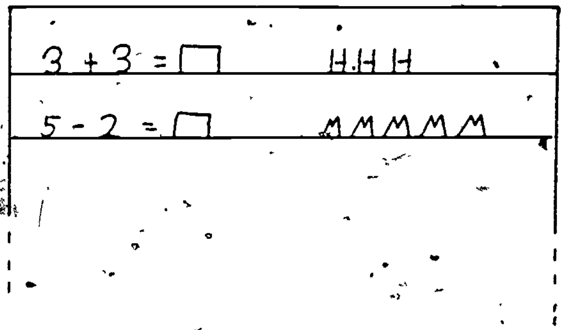
(mastery indicative of readiness obj. 3 or obj.4 or both)

No Worksheet

S_{34}

Materials: Worksheet, pencil.

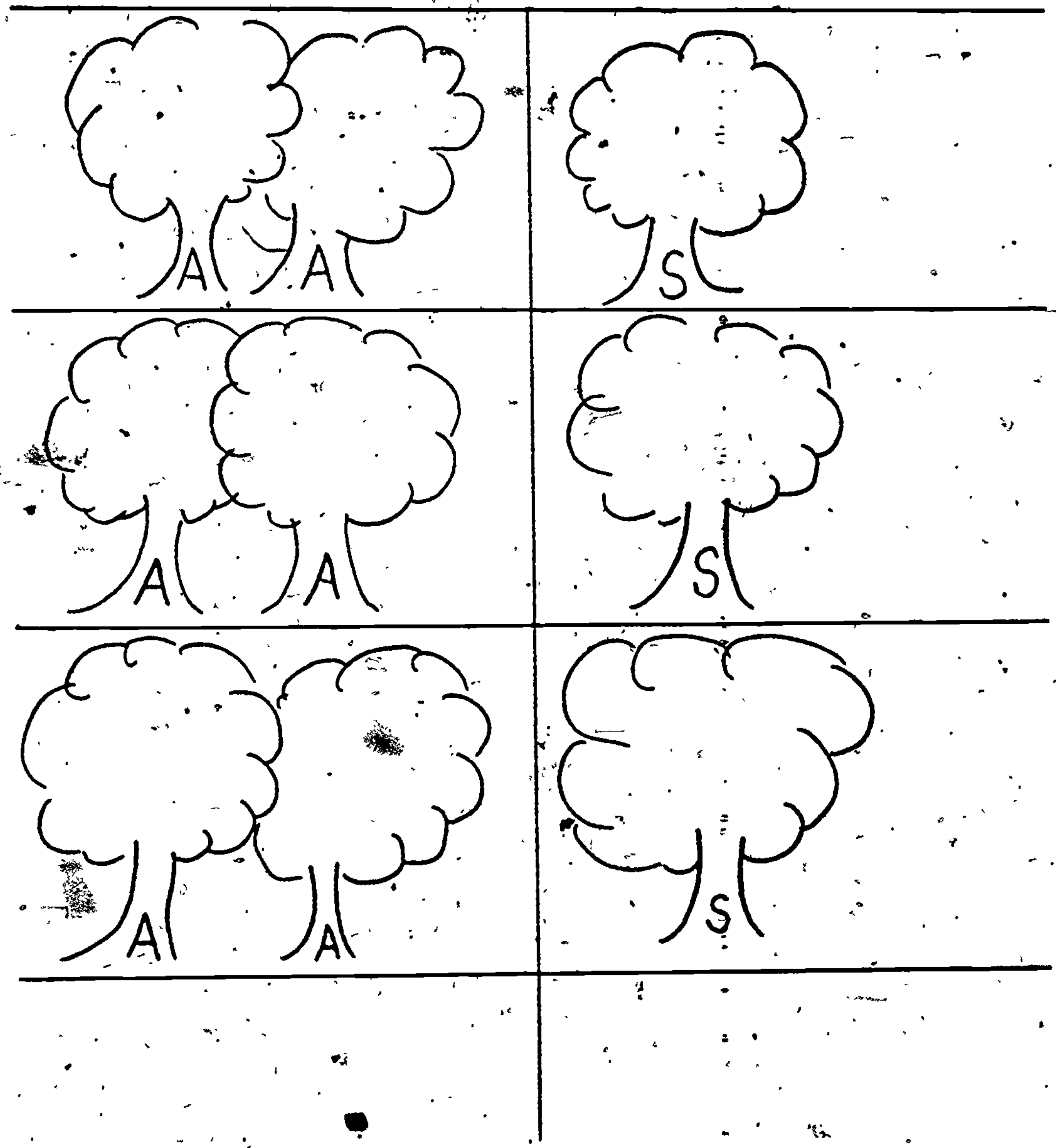
Find the answer. mark off letters or add letters to show the number sentence:



Worksheet S-34

Obj. 3.4 (AS 7)
Sheet # CP-34
Name ____________
Date ____________

NAME ____________



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Obj. 3,4 (AS 7)
Sheet # S-34
Name ____________
Date ____________

NAME ____________

3 + 3 =	☐	M M M
5 - 2 =	☐	F F F F F
6 + 1 =	☐	C C C C C C
4 - 1 =	☐	T T T T
4 + 4 =	☐	P P P P
5 - 3 =	☐	E E E E E
5 + 2 =	☐	S S S S S
7 - 5 =	☐	Q Q Q Q Q Q Q

APPENDIX D

POSTTEST

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Answers
Stimulus: Verbal number sentences
Response: Verbal answer

Say: Tell me the answers to these number sentences:

I. a. 2 + 2 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

b. 3 + 2 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

c. 4 + 5 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

d. 7 + 6 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

e. 3 - 1 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

f. 5 - 2 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

g. 6 - 3 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

h. 12 - 4 = ____ correct incorrect________immediate delayed
counted on counted fingers no response

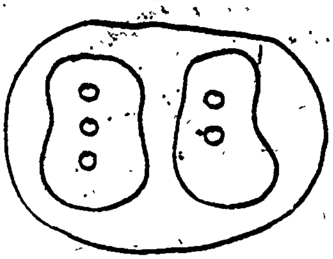
i. 3 + ____ = 5 correct incorrect________immediate delayed
(what) counted on counted fingers no response

j. 7 - ____ = 5 correct incorrect________immediate delayed
(what) counted on counted fingers no response

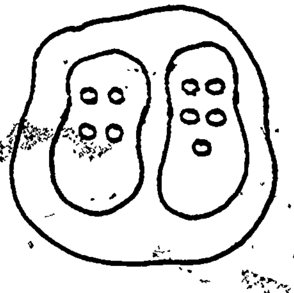
Production of Number Sentences
Stimulus: Picture
Response: Verbal Number Sentence

Instructions: Point to each of the pictures in turn. Ask the child to tell you a number sentence for the picture.

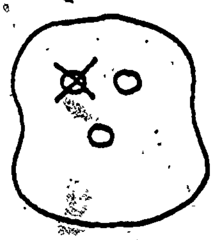
II. 1. Verbal number sentence: Correct incorrect_______________no response



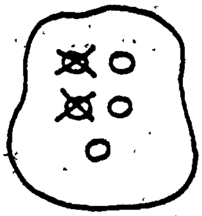
2. Verbal number sentence correct incorrect_______________no response



3. Verbal number sentence correct incorrect_______________no response



4. Verbal number sentence correct incorrect_______________no response



Answers & Production of Number Sentences
Stimulus: Verbal story problems
Response: Verbal answer & verbal written number sentence
Materials: Counters, blank paper

Say: I am going to tell you some stories and ask you some questions about the stories. I want you to tell me the answers to the questions. You may use the counters or your fingers if you wish.

III. Story 1. Pretend you have 2 pennies and you found 2 more pennies. How many pennies would you have?

a. Record answer
Answer: correct incorrect________counters immediate delayed no response

b. Say: Can you tell me a number sentence for this story?
Response:
Verbal Number Sentence: correct incorrect________________ delayed no response

Story 2. ____(child's name)____ found 3 acorns on the playground. He found 2 more acorns on the steps. How many acorns did he find?

a. Record answer
Answer: correct incorrect________counters immediate delayed no response

b. Say: Can you write a number sentence for this story?
Response:
Written Number Sentence: correct incorrect________________ no response

Story 3. Five children were in line to go to lunch. Two children left to wash their hands. How many children were still in line?

a. Record answer
Answer: correct incorrect________counters immediate delayed no response

b. Say: Can you tell me a number sentence for this story?
Response:
Verbal Number Sentence: correct incorrect________________ no response

Answers and Production of Number Sentences (cont.)

Story 4. There were 3 birds in a tree. One bird flew away. How many birds were left?

a. Record answer

Answer: correct incorrect ________ counters immediate delayed
no response

b. Say: Can you write a number sentence for this story?

Response:

Written Number Sentence: correct incorrect ____________
no response

Story 5. Let's pretend you have 3 pennies. You want to buy chocolate milk that costs 5 pennies. How many more pennies do you need?

a. Record answer

Answer: correct incorrect ________ counters immediate delayed
no response

b. Say: Can you tell me a number sentence for this story?

Response:

Verbal Number Sentence: correct incorrect ____________
no response

Story 6. Let's pretend you have 7 cookies. How many cookies must you eat so that there are only 4 cookie left?

a. Record answer

Answer: correct incorrect ________ counters immediate delayed
no response

b. Say: Can you write a number sentence for this story?

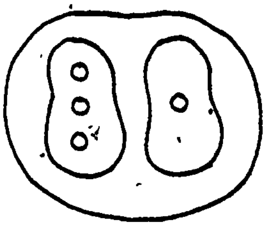
Response:

Written Number Sentence: correct incorrect
no response

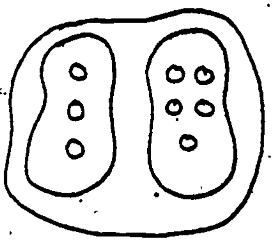
Production of Number Sentences
Stimulus: Picture
Response: Written number sentence

Instructions: Point to each of the pictures in turn. Ask the child to write a number sentence for the picture

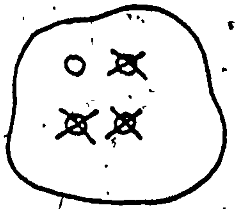
IV 1.



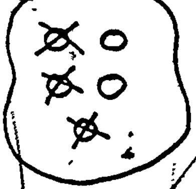
2.



3.



4.



Answers and Interpretation of Number Sentences
Stimulus: Written Number Sentence
Response: Written Answer, Objects Interpretation
Materials: Objects

V Instructions: Show the child each of the following number sentences. Ask the child to write the answer in the blank. Say: Show me how you would find the answer to this number sentence if you forgot it. Use these objects or your fingers.

3 + 2 = ______

1. a. Written answer: delayed immediate no response counted on counted fingers
   b. Interpretation: correct incorrect no response
   method used ____________________

7 + 6 = ______

2. a. Written answer: delayed immediate no response counted on counted fingers
   b. Interpretation: correct incorrect no response
   method used ____________________

3 - 1 = ______

3. a. Written answer: delayed immediate no response counted on counted fingers
   b. Interpretation: correct incorrect no response
   method used ____________________

Answers and Interpretation of Number Sentences (cont.)

12 - 4 = ______

4. a. Written answer: delayed immediate no response counted on counted fingers

b. Interpretation: correct incorrect no response

method used ______

Interpretation of Number Sentences
Stimulus: Verbal Number Sentence
Response: Picture, Objects
Materials: Picture sheet, objects

VI. Instructions: Give the child the picture sheet. Say: I am going to tell you some number sentences. I want you to point to the picture that shows the number sentence.

a. 2 + 2 correct incorrect________________no response

b. 3 + 2 correct incorrect________________no response

c. 3 - 1 correct incorrect________________no response

d. 5 - 2 correct incorrect________________no response

VII. Instructions: Give the child a set of objects ~~and two number lines.~~ Say: I'm going to tell you some number sentenses. Show how you could find the answer if you forgot it. Use these objects or your fingers.

a. 3 + 2 Interpretation: correct incorrect

method used________________________________

b. 5 - 2 Interpretation: correct incorrect

method used________________________________

Answers & Interpretation of Number Sentences
Stimulus: Written Number Sentence
Response: Verbal answer, Object Interpretation
Materials: Objects

VIII Instructions: Show the child each of the following number sentences. Ask the child to tell you the answer. (Record response) Say: Show me how you would find the answer to this number sentence if you forgot it. Use these objects or your fingers.

2 + 2 = ________

1. a. Verbal answer: correct incorrect ____ delayed immediate no response

   counted on counted fingers other ________________

   b. Interpretation: correct incorrect no response

   method used ____________________________

4 + 5 = ________

2. a. Verbal answer: correct incorrect ____ delayed immediate no response

   counted on counted fingers other ________________

   b. Interpretation: correct incorrect no response

5 - 2 = ________

3. a. Verbal answer: correct incorrect ____ delayed immediate no response

   counted on counted fingers other ________________

   b. Interpretation: correct incorrect no response

   method used ____________________________

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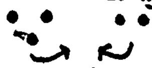
Production of Number Sentences
Stimulus: Objects
Response: Verbal & Written Number Sentences

	DO	SAY
IX.	1. Place a set of 3 objects and a set of 2 objects on the table. Move together but do not mix.	1. Tell me a number sentence for what I did.



Spoken number sentence: correct incorrect________________no response

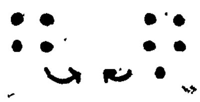
2. Place a set of 5 objects on the table. Remove two objects. — 2. Write a number sentence for what I did.

Written number sentence: correct incorrect________________no response

3. Place a set of 6 objects on the table. Remove a set of 3 objects. — 3. Tell me a number sentence for what I did

Spoken number sentence: correct incorrect________________no response

4. Place a set of 4 objects and a set of 5 objects on the table. Move together but do not mix. — Write a number sentence for what I did.



Written number sentence: correct incorrect________________no response

APPENDIX E

LEVEL A OBJECTIVES FOR THE INDIVIDUALLY

PERSCRIBED INSTRUCTION (IPI)

MATHEMATICS PROGRAM

Level [illegible]

NUMERATION/PLACE VALUE
PART I

1. Given pictures of objects, the student identifies the smallest object, the largest object, and objects of the same size. Given pictures of two sets of objects, the student identifies which set has more, which has less, or if the sets have the same number of elements.

2. Given pictures of two sets, the student pairs [illegible] to determine whether one set has more elements than, less elements than, or as many elements as the other.

3. Given a picture of objects, the student indicates whether the objects are ordered by size. Given a picture of several sets, the student indicates whether the sets are ordered by the number of elements in each set.

4. The student says the number names in order.
   LIMIT 1 through 5
   NO BOOKLET

5. Given a picture of a set, the student determines the number of elements in the set by counting aloud.
   LIMIT 1 through [illegible]

6. Given a numeral, the student says the number name; given a spoken number name, the student identifies the numeral. Given a picture of a set, the student identifies the numeral that indicates how many elements are in the set.
   LIMIT 1 through 5

7. Given a numeral, the student identifies or constructs a set of objects having the given number of elements.
   LIMIT 1 through 5

8. Given a spoken number name, the student writes the numeral. Given a picture of a set, the student writes the numeral that indicates how many elements are in the set.
   LIMIT 1 through 5

9. Given two numerals, the student identifies the numeral that names the greater or lesser number. Given numerals in random order, the student writes them in ascending order.
   LIMIT 1 through 5

PART II

10. The student says the number names in order.
    LIMIT 1 through 10
    NO BOOKLET

11. Given a picture of a set, the student determines, by counting aloud, the number [illegible] in the set [illegible]

12. Given a numeral, the student says the number name; given a spoken number name, the student identifies the numeral. Given a picture of a set, the student identifies the numeral that indicates how many elements are in the set.
    LIMIT 0 through 9

13. Given a spoken number name, the student writes the numeral. Given a picture of a set, the student writes the numeral that indicates how many elements are in the set.
    LIMIT 0 through 9

14. Given two numerals, the student identifies the numeral that names the greater or lesser number. Given numerals in random order, the student writes them in ascending order.
    LIMIT 0 through 9

15. Given pictures of two sets, the student writes the numeral that indicates how many elements are in each set. He identifies which set has more elements, which set has less elements, or if the two sets are equivalent.
    LIMIT 0 through 9

ADDITION/SUBTRACTION
PART I

1. Given pictures of two sets, the student makes (by drawing more or crossing out objects) the two sets equivalent.
   LIMIT maximum of 9 elements

2. Given pictures of two sets, the student draws a picture showing the union of these sets.
   LIMIT maximum of 9 elements in the union set

3. Given a picture of a set, the student shows (by crossing out objects) the removal of a subset.
   LIMIT maximum of 9 elements

4. Given a picture illustrating the union of sets or the removal of a subset, the student writes the number sentence.
   LIMIT sums through 5

5. Given a picture illustrating the union of sets or the removal of a subset, the student writes the number sentence.
   LIMIT sums from 6 through 9

6. Given pictures of two sets, the student illustrates the union of the sets and writes the number sentence. Given directions to take away a specified number of objects and a picture of a set of objects, the student illustrates the removal of a subset and writes the number sentence.
   LIMIT sums through 9

7. Given an addition or subtraction sentence, the student illustrates the union of sets or the removal of a subset.
   LIMIT sums through 9

8. Given a number line illustrating a number sentence, the student identifies whether addition or subtraction is indicated.
   LIMIT sums through 9

9. Given a number line illustrating a number sentence, the student writes the addition or subtraction sentence.

10. Given a number line, the student uses it to illustrate and complete addition and subtraction sentences.
    LIMIT sums through 5

11. Given a number line, the student uses it to illustrate and complete addition and subtraction sentences.
    LIMIT sums through 9

PART II

12. Given an addition or subtraction sentence in the form $a+b=\square$ or $a-b=\square$, the student reads and completes the sentence.
    LIMIT sums through 9

13. Given an addition or subtraction example written in vertical form, the student reads and completes the example.
    LIMIT sums through 9

14. Given an addition sentence, the student writes two subtraction sentences using the same numbers.
    LIMIT sums through 9

15. Given a number sentence in the form $a+\square=c$ or $\square+b=c$, the student completes the sentence.
    LIMIT sums through 9

16. Given a number sentence in the form $a=\square+\triangle$, the student completes the sentence in several ways.
    LIMIT sums through 9

17. Given a sum and a picture of a set or a number line, the student completes several addition and subtraction sentences for the specified sum.
    LIMIT sums through 9

FRACTIONS

1. Given a picture of a partitioned shape, the student indicates whether or not the parts are equivalent.
   LIMIT two, three, or four parts

2. Given a picture of a shape partitioned into equivalent parts, the student writes the numeral that indicates the number of parts.
   LIMIT two, three, or four parts

3. Given a picture of a whole object and fractional parts of that object, the student identifies one-half, one-fourth, and a whole.

MONEY

1. Given a collection of U.S. coins and the name of a specified coin, the student matches the specified coin with a picture of either of its faces.

TIME

1. The student says the names of the days of the week in order.
   NO BOOKLET

APPENDIX F

POSTTEST SCORES

APPENDIX F

Table 14 Posttest Raw Scores of Not Ready Subjects

Subject	Delayed Symbolization Group: No. of number sentences on posttest correctly interpreted A	S	T	produced A	S	T	answered A	S	T	Total A	S	T	Subject	Immediate Symbolization Group: No. of number sentences on posttest correctly interpreted A	S	T	produced A	S	T	answered A	S	T	Total A	S	T
1	7	6	13	8	8	16	9	9	18	24	23	47	1	7	4	11	7	5	12	10	8	18	24	17	41
2	7	6	13	5	8	13	9	8	17	21	22	43	2	6	6	12	7	6	13	9	8	17	22	20	42
3	7	5	12	7	7	14	7	7	14	21	19	40	3	5	4	9	7	4	11	8	6	14	20	14	34
4	6	4	10	2	0	2	8	8	16	16	11	27	4	7	5	12	2	3	5	8	8	16	17	16	33
5	7	6	13	8	6	14	10	6	16	25	18	43	5	4	4	8	5	5	10	9	8	17	18	17	35
6	7	6	13	7	8	15	9	7	16	23	21	44	6	7	5	12	8	4	12	10	9	19	25	18	43
7	7	4	11	5	4	9	9	5	14	21	13	34	7	6	1	7	4	4	8	10	8	18	20	13	33
8	7	4	11	7	5	12	9	2	11	23	11	34	8	6	3	9	3	0	3	7	1	8	16	4	20
9	6	4	10	2	2	4	6	5	11	14	11	25	9	6	5	11	5	3	8	7	5	12	18	13	31
10	7	6	13	7	6	13	9	5	14	23	17	40	10	6	3	9	4	5	9	8	4	12	18	12	30
11	2	2	4	1	0	1	4	3	7	7	5	12	11	1	0	1	0	0	0	4	2	6	5	2	7

APPENDIX F

Table 13 Posttest Raw Scores of Ready Subjects

Delayed Symbolization Group

Subject	No. of number sentences on posttest correctly interpreted			produced			answered			Total		
	A	S	T	A	S	T	A	S	T	A	S	T
1	6	6	12	8	7	15	10	7	17	24	20	44
2	7	3	10	7	8	15	7	7	14	21	18	39
3	7	5	12	7	7	14	8	8	16	22	20	42
4	7	6	13	6	7	13	8	8	16	21	21	42
5	7	6	13	8	8	16	10	8	18	25	22	47
6	7	3	10	7	7	14	9	2	11	23	12	35
7	7	6	13	8	8	16	9	8	17	24	22	46
8	6	6	12	7	7	14	9	9	18	22	22	44

Immediate Symbolization Group

Subject	No. of number sentences on posttest correctly interpreted			produced			answered			Total		
	A	S	T	A	S	T	A	S	T	A	S	T
1	7	6	13	8	7	15	9	7	16	24	21	45
2	7	6	13	8	8	16	9	7	16	24	21	45
3	7	6	13	8	8	16	9	9	18	24	23	47
4	5	5	10	5	6	11	8	6	14	18	17	35
5	7	6	13	8	8	16	10	7	17	26	21	47
6	7	6	13	7	8	15	7	8	15	21	22	43
7	7	6	13	8	8	16	10	9	19	25	23	48
8	7	6	13	8	7	15	10	6	16	26	19	45